Problem Sets

Graph Theory - MATH 247

April 5, 2021

Problem sets due at 5pm of the day indicated via the Canvas site. *Please note that the first printing of the text has some typographical errors. I'll try to alert these to you when I know about them.*

1. Due Wednesday, March 3
   Read: Syllabus, Thoughts on Homework
   Read: textbook's Preface, Prologue, Chapter 1
   Turn in (via Canvas site): Problems 2, 7, 9, 12, 19
   Note: In drawing the graphs from these problems, please draw such that structural characteristics are easily visible.

2. Due Wednesday, March 10
   Read: Chapter 2; Handout on Mathematical Induction (see the Google folder); and first page of "Vertex-coloring edge-weightings: Towards the 1-2-3-conjecture" by Kalkowski, Karoński and Pfender
   Turn in: Problems 2, 7, 8, 13, 19

   Also do: Problem 6, Prove Theorem 2.3 using induction on the number of distinct elements in the set

3. Due Wednesday, March 17
   Read: Chapter 3 and "Merlin's Magic Square" by Don Pelletier (which is available in the Google folder)
   Turn in: Problems 5, 13, 17, 21, 23

   Install software on a laptop so that you can use LaTeX. See the instructions on the course webpage. If this is not possible, please let me know. You will be asked to use LaTeX to typeset at least one problem from each future problem set, including this one.

   Also do: Problem 22; 25; Prove that every graph with no odd cycles is bipartite using induction on the number of vertices.

4. Due Wednesday, March 24
   Read: Chapter 4
   Turn in: Problems 5, 10, 11(a),(c), 14. (At least one of these problems must be typeset using LaTeX.)

   Also do: 4, 11(b), 11(d), 15, find the Prüfer code of a given labelled tree, given a Prüfer code draw the labelled tree

5. Due Friday, April 2
   Read: Chapter 5
   Turn in: 2, 8, 13, 17

   Also do: Apply Dijkstra's algorithm to a weighted graph; know the definition of De Bruijn graphs and what the first few look like; State and prove a characterization of Eulerian digraphs (recall the definition of strongly-connected);

   This article about de Bruijn graphs and genome assembly is interesting:
   https://www.ncbi.nlm.nih.gov/pmc/articles/PMC5531759/

6. Due ??
   Read: Chapter 6
   Turn in: 6, 9, 12, Prove Theorem 6.3
   Also do: 10; Prove that Theorem 6.2 is a corollary of Theorem 6.3; Characterize when the graph $K_{n_1,n_2,\ldots,n_p}$ is hamiltonian (and justify the characterization, of course!); Prove that the Petersen graph is not hamiltonian; Prove that Q_4 is hamiltonian without referring to a drawing and without listing out the order one visits all 16 vertices; Give an example of a graph on 6 vertices that shows that the greedy approach to TSP can be arbitrarily bad

7. Due ??
   Read: Chapter 7
   Turn in: 1, 4, 9, 15 *(For problem 15: the first printing of the text doesn't include the edges u_2u_3 and v_2v_3, so please insert these.)*, and 16