

Graph Theory - MATH 247

Exam 3

Name:

Honor Code Pledge

Signature

Directions: Please complete all but 1 problem. There is a time limit of 3 hours.

1. Use the Four Color Theorem to prove that every planar graph decomposes into two bipartite graphs. What further can you say if the condition color-critical is also imposed?
2. Use Euler's Formula and the fact that the Petersen graph has girth 5 to show that the Petersen graph is nonplanar.
3. Let G be a maximal planar graph. Prove that G^* , the dual graph of G , is 2-edge-connected and 3-regular.
4. Prove or disprove: For every graph G , $\chi(G) \leq n(G) - \alpha(G) + 1$.
5. The wheel graph, W_n , is the join of a single vertex and C_n . For example, W_3 is isomorphic to K_4 . Determine $\chi(W_n)$.
6. Show an embedding of K_6 on the torus. Heawood's formula states that $\chi(G) \leq \lfloor \frac{7 + \sqrt{1 + 48\gamma}}{2} \rfloor$, where γ denotes the genus of the surface. Is this formula useful in this scenario? Why, or why not?
7. A kitchen sink draws water from two tanks according to the network of pipes with capacities per unit time shown below. Find the maximum flow. Prove that your answer is optimal.

