Graph Theory - MATH 247

Exam 2

Name:
Honor Code Pledge:

Signature:

Directions: Please complete all but 1 problem. There is a time limit of 3 hours.

1.
   - Given the vertex-labelled tree in Figure 1 determine its Prüfer code.
   - Given the following Prüfer code $(1, 2, 3, 3, 2, 1)$, determine the tree which corresponds to it.

2. Find a minimum weight spanning tree of the graph in Figure 2.

3. Use Cayley's Formula to prove that the graph obtained from K_n by deleting an edge has $(n-2)n^{n-3}$ spanning trees. [West]

4. Determine the minimum size of a maximal matching in the cycle C_n. [West] Do the same for C_{2k} with one chord joining two antipodal vertices (i.e. an edge joining two vertices at distance k on the cycle). [Schmitt]

5. An algorithm to greedily build a large independent set S iteratively selects a vertex of minimum degree in the remaining graph, adds it to S, and deletes it and its neighbors from the graph. Use induction on the number of vertices to prove that this algorithm produces an independent set of size at least $\sum_{v \in V(G)} \frac{1}{d_G(v)+1}$ in a simple graph G. [West]

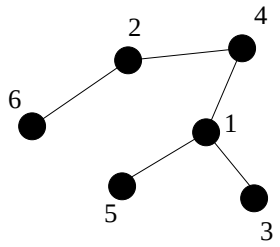

Figure 1: A vertex-labelled tree

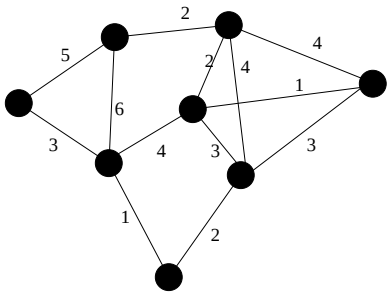

Figure 2: A weighted graph

6. One day the president of Poland, who speaks only Polish, decides to visit the Middlebury Language Schools, where ten languages, which we label $L_1, \ldots, L_{10}$, none of which is Polish, are spoken. He decides to give a speech on the importance of mathematics in a liberal arts education to all of the Schools simultaneously and thankfully he has brought along ten translators $T_1, \ldots, T_{10}$ each of whom speaks Polish and some three of the ten languages taught at Middlebury. The Language Schools determine that for each of the languages taught there are three translators that can speak it. Can it be arranged so that his speech can be simultaneously translated to each of the Schools or is there a scenario that will result in sadness for one of the Schools? [Schmitt]

7. Let G be a 20-vertex graph without isolated vertices. Suppose the size of a maximum independent set is 8 and the size of a maximum matching is 9. What can you say about each of the following:

   - the minimum size of a a vertex cover,
   - the minimum size of an edge cover?