Graph Theory - MATH 247

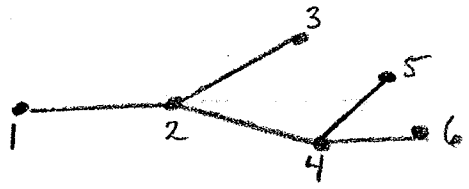
Exam 2

Name:
Honor Code Pledge

Signature

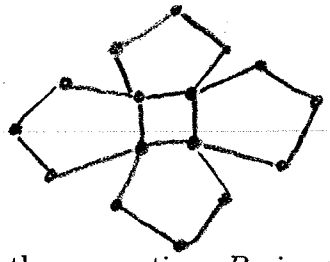
Directions: Please complete all but 1 problem. There is a time limit of 2 hours.

1. • Given the following tree determine its Prüfer code.



   • Given the following Prüfer code, $(1, 2, 2, 3, 3, 3)$ determine the tree which corresponds to it.

2. Count the number of spanning trees in the graph given below.



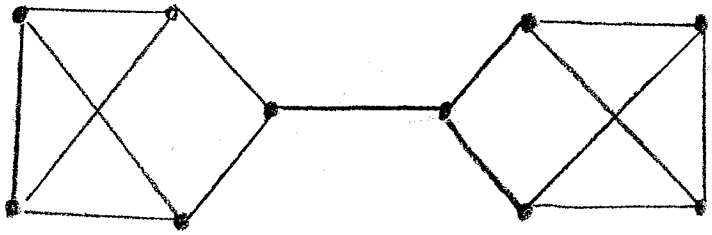
3. Prove that the path on n vertices, P_n, is graceful. (Do not just claim that a path is a caterpillar, and therefore graceful. Rather, give the labelling that demonstrates that P_n is graceful.)

4. Let G be a 3-regular graph with at most two cut-edges. Prove that G has a 1-factor.

5. There are five cities in a network. The cost of building a road directly between i and j is the entry $a_{i,j}$ in the matrix below. An infinite entry indicates that there is a mountain in the way and the road cannot be built. Determine the least cost of making all the cities reachable from each other.

$$\begin{pmatrix} 0 & 3 & 5 & 11 & 9 \\ 3 & 0 & 3 & 9 & 8 \\ 5 & 3 & 0 & \infty & 10 \\ 11 & 9 & \infty & 0 & 7 \\ 9 & 8 & 10 & 7 & 0 \end{pmatrix}$$

6. Prove that a cubic graph which contains a bridge is not decomposable into three 1-factors. (To help you think about this problem, I give the graph below as an example of a cubic graph with a bridge. Proving the statement for this example is not sufficient.)



7. Prove that a graph G is k-connected if and only if $G \vee K_r$ (the join of G and K_r) is $k + r$-connected.

8. Demonstrate that Hall's condition holds for the bipartite graph given below without demonstrating the matching that saturates X.