Graph Theory - MATH 247 - Fall '11

Exam 1

Name:
Honor Code Pledge:

Signature:

Directions: This exam is a closed-book, closed-notes exam. Please complete all but 1 problem. Each problem is worth 10 points. Please complete the problems on separate paper and place them in order upon stapling to this sheet.

1. Determine if the following sequences are graphic. If the sequence is not graphic, then indicate why. If the sequence is graphic, give a graph that realizes the sequence.

   - $(4, 4, 4, 4, 2, 2)$
   - $(9, 3, 3, 3, 3, 3, 3, 3)$

2. Decompose K_7 into 7-cycles. Decompose K_9 into 9-cycles. How many cycles would be in a 15-cycle decomposition of K_{15}?

3. Prove that if v is a cut-vertex of a simple graph G, then v is not a cut-vertex of $\overline{G}$. [West]

4. Prove that in every connected simple graph with at least two vertices there are two vertices of equal degree. [West]

5. Prove that every non-trivial tree has at least two maximal independent sets. [West]

6. Use induction on n to prove that if $d_1, \ldots, d_n$ are nonnegative integers and $\sum d_i$ is even, then there is an n-vertex graph with vertex degrees $d_1, \ldots, d_n$. (Note: we are **not** making the assumption of simple.)[West]

7. The scene is the estate of the well-to-do billionaire Count Van Diamond. He has just been murdered, and Juan Bomb, the internationally known detective and part-time graph theorist, has been called in to investigate. The butler claims he saw the gardener enter the pool room (where the murder took place) and then, shortly after, leave that room by the same door. The gardener, however, says that he cannot be the man whom the butler saw, for he entered the house, went through each door exactly once and then left the house. Juan Bomb checks the floor plan (given below). Within a matter of hours (or maybe minutes), he declares the case solved. Who killed the Count? [Chartrand]