Calculus II - Exam 3 - Fall 2016

November 17, 2016

Name:
Honor Code Statement:

Directions: Upon completion of the examination and prior to its submission, please write and sign the Honor Code. **Justify** all answers/solutions. Make sure to indicate the test or theorem that you use. Calculators are not permitted, and all electronic devices should be off. Good luck!

1. [5 points] Theorem 6 of Chapter 11 states: If the series $\sum_{n=1}^{\infty} a_n$ is convergent, then $\lim_{n\to\infty} a_n = 0$. The harmonic series shows that the converse to this statement is false, i.e. $\lim_{n\to\infty} \frac{1}{n} = 0$ but $\sum_{n=1}^{\infty} \frac{1}{n}$ is divergent. Give another example that shows that the converse is false.

2. [5 points] For what values of r is the *sequence* $\{r^n\}$ convergent? For those values of r for which the sequence is convergent, state the limit.

3. [5 points] State the Monotonic Sequence Theorem. Give an example of a sequence to which the theorem applies and prove that the hypotheses of the theorem apply to the example that you give.

4. [5 points] What does the following geometric series converge to, $1 + 3/10 + 9/100 + 27/1,000 \ldots$?

5. [8 points each] **Complete four of the following five. Indicate which problem you omit with a slash through it.** For each of the following series, determine whether or not the series converges. If the series contains any negative terms, please test for absolute convergence. State the theorem (i.e. test) that you use to draw your conclusion.

(a)

$$\sum_{n=2}^{\infty} \frac{\ln n}{n^2}$$

(b)

$$\sum_{n=1}^{\infty} \frac{\sqrt[3]{n}}{\sqrt{n^3 + n + 1}}$$

(c)

$$\sum_{n=1}^{\infty} \frac{n\cos(n\pi)}{2^n}$$

(Hint: re-express the n^{th}-term of the series before applying any test.)

(d)

$$\sum_{n=1}^{\infty} \left(\frac{n}{\ln n}\right)^n$$

(e)

$$\sum_{n=1}^{\infty} \frac{(-3)^n}{(2n+1)!}$$

6. [8 points] Find the value of c such that

$$\sum_{n=0}^{\infty} e^{nc} = 10.$$

7. [8 points] Explain how the figure given proves the provided equations. [R. Nelsen, Proofs without words]

$$\sum_{n=0}^{\infty} ar^n = \frac{a}{1-r}$$

$$\frac{a + ar + ar^2 + ar^3 + \cdots}{1/r} = \frac{ar}{1-r}$$