MULTIVARIABLE CALCULUS
EXAM 3
SPRING 2018

Name:
Honor Code Statement:

Directions: Complete all problems. Justify all answers/solutions. Electronic devices, books, and notes are not permitted. Please turn off cell phones and other devices. The last two pages contains formulas. Best of luck.

(1) [10 points] Compute the following iterated integral:

$$\int_{-5}^{5}\int_{-1}^{2}(5-|y|)dx\ dy$$

Date: May 18, 2018.

(2) [10 points] Evaluate $\iint_D 3y\ dA$, where D is the region bounded by $xy^2 = 1, y = x, x = 0$ and $y = 3$. BEFORE proceeding, please make a sketch of the region! Graph paper is available.

(3) [5 points] In the previous question you evaluated an iterated integral. Write an equivalent iterated integral with the order of integration reversed. (You need not evaluate this new integral.)

(4) [5 points] Each of the following statements is false. Correct the statement (in as minimal a way as possible) and without simply negating the solution.

- If $\mathbf{x}$ and $\mathbf{y}$ are two one-one parametrizations of the same curve and $\mathbf{F}$ is a continuous vector field, then $\int_{\mathbf{x}} \mathbf{F} \cdot ds = \int_{\mathbf{y}} \mathbf{F} \cdot ds$.

- Suppose that $f(x) > 0$ for all x. Let $\mathbf{F} = f(x)\mathbf{i}$. If C is the vertical line segment from $(0, 0)$ to $(0, 3)$, then $\int_C \mathbf{F} \cdot ds > 0$.

(5) [10 points] Evaluate the following integral where D is the region in the first quadrant bounded by the hyperbolas $x^2 - y^2 = 1, x^2 - y^2 = 4$ and the ellipses $x^2/4 + y^2 = 1, x^2/16 + y^2/4 = 1$. Graph paper is attached to help you sketch this region. To do this integral, I recommend using the change of variables $u = x^2 - y^2$ and $v = x^2/4 + y^2$. (Hint: Don't work too hard to find the limits of integration, stare at the equations you have for a bit.)

$$\iint_D \frac{xy}{y^2 - x^2} dA$$

(6) [10 points] Calculate the following scalar line integral where $\mathbf{x}(t) = (\cos 4t, \sin 4t, 3t)$ and $0 \leq t \leq 2\pi$.

$$\int_{\mathbf{x}} 3x + xy + z^3 ds$$

(7) [10 points] Use Green's theorem to find the area enclosed by the hypocycloid $\mathbf{x}(t) = (\cos^3(t), \sin^3(t))$, for $0 \le t \le 2\pi$. Hint: the area is given by $\iint_D dy\, dx$.

(8) [5 points] Suppose that the temperature at a point in the cube

$$W = [-1, 1] \times [-1, 1] \times [-1, 1]$$

varies in proportion (with constant of proportionality k) to the square of the point's distance from the origin. Set up, but do not evaluate, an integral that gives the average temperature of the cube.

(9) [10 points] Use Green's theorem to find the work done by the vector field $\mathbf{F} = x^2y\mathbf{i} + (x + y)y\mathbf{j}$ in moving a particle from the origin along the y-axis to the point $(0, 1)$, then along the line segment from $(0, 1)$ to $(1, 0)$, and then from $(1, 0)$ back to the origin along the x-axis.

Change of coordinates

Cylindrical to Cartesian:

$$x = r\cos\theta,\ y = r\sin\theta, z = z$$

Cartesian to cylindrical:

$$r^2 = x^2 + y^2,\ \tan(\theta) = \frac{y}{x},\ z = z$$

Spherical to Cartesian:

$$x = \rho\sin\varphi\cos\theta,\ y = \rho\sin\varphi\sin\theta,\ z = \rho\cos\varphi$$

Cartesian to spherical:

$$\rho^2 = x^2 + y^2 + z^2,\ \tan(\varphi) = \sqrt{x^2+y^2}/z,\ \tan(\theta) = \frac{y}{x}$$

Spherical to cylindrical:

$$r = \rho\sin(\varphi),\ \theta = \theta,\ z = \rho\cos(\varphi)$$

Cylindrical to spherical:

$$\rho^2 = r^2 + z^2,\ \tan(\varphi) = r/z,\ \theta = \theta$$

Change of variables in triple integrals:

$$\int\int\int_W f(x,y,z)dxdydz = \int\int\int_{W^*} f(x(u,v,w), y(u,v,w), z(u,v,w))|\frac{\partial(x,y,z)}{\partial(u,v,w)}|dudvdw$$

Volume elements:

$$dV = dx\ dy\ dz\ Cartesian$$
$$dV = r\ dr\ d\theta\ dz\ Cylindrical$$
$$dV = \rho^2\sin\varphi\ d\rho\ d\varphi\ d\theta\ spherical$$
$$dV = |\frac{\partial(x,y,z)}{\partial(u,v,w)}|du\ dv\ dw\ general$$

Trigonometric Identities

Addition and subtraction formulas

- $\sin(x+y) = \sin x \cos y + \cos x \sin y$
- $\sin(x-y) = \sin x \cos y - \cos x \sin y$
- $\cos(x+y) = \cos x \cos y - \sin x \sin y$
- $\cos(x-y) = \cos x \cos y + \sin x \sin y$
- $\tan(x+y) = \frac{\tan x + \tan y}{1 - \tan x \tan y}$
- $\tan(x-y) = \frac{\tan x - \tan y}{1 + \tan x \tan y}$

Double-angle formulas

- $\sin(2x) = 2 \sin x \cos x$
- $\cos(2x) = \cos^2 x - \sin^2 x = 2\cos^2 x - 1 = 1 - 2\sin^2 x$
- $\tan(2x) = \frac{2 \tan x}{1 - \tan^2 x}$

Half-angle formulas

- $\sin^2 x = \frac{1 - \cos(2x)}{2}$
- $\cos^2 x = \frac{1 + \cos(2x)}{2}$

Others

- $\sin A \cos B = \frac{1}{2}[\sin(A-B) + \sin(A+B)]$
- $\sin A \sin B = \frac{1}{2}[\cos(A-B) - \cos(A+B)]$
- $\cos A \cos B = \frac{1}{2}[\cos(A-B) + \cos(A+B)]$

Pythagorean and reciprocal identities

- If you don't know these, then get a tattoo.