MULTIVARIABLE CALCULUS
EXAM 2
FALL 2025

Name:
Honor Code Statement:

Directions: You may **OMIT** one problem – do so by putting a slash through it. Justify all answers/solutions. Each problem is worth 10 points. Calculators/notes/texts/cell-phones are not permitted – the only permitted item is a writing utensil. Page 288 of the text is photocopied at the end of the exam. The exam is proctored by permission of the Dean of the Faculty. Best of luck.

(1) Find the arc length function for the curve $\mathbf{x}(t) = (t, t^2 - \frac{1}{8}\ln(t))$ with initial starting point $(1, 1)$. (Hint: there will be a perfect square underneath the radical sign.)

Date: November 13, 2025.

(2) A moving particle starts at an initial position $\mathbf{x}(0) = (1, 0, 0)$ with initial velocity $\mathbf{v}(0) = \mathbf{i} - \mathbf{j} + \mathbf{k}$. Its acceleration is $\mathbf{a}(t) = 4t\mathbf{i} + 6t\mathbf{j} + \mathbf{k}$. **Find** its velocity $\mathbf{v}(t)$ at time t. **Find** its position $\mathbf{x}(t)$ at time t. **Write one sentence** that describes the position of the particle as t approaches infinity.

(3) Calculate the flow line $\mathbf{x}(t)$ of the vector field $\mathbf{F}(x, y, z) = 2\mathbf{i} - 3y\mathbf{j} + z^3\mathbf{k}$ at the point $\mathbf{x}(0) = (3, 5, 7)$.

(4) For the same vector field as in the previous problem: calculate the divergence of $\mathbf{F}$ at this point, and calculate the curl of $\mathbf{F}$ at this point.

(5) Find the first- and second-order Taylor polynomials for the function

$$f(x,y) = ye^{3x}$$

at the point $\mathbf{a} = (0,0)$. Use this second order Taylor polynomial to estimate $f(0.1, 0.1)$.

(6) Find the one critical point of the following function. Then use the second derivative test for functions of two variables to determine the nature of this critical point.

$$f(x, y, z) = x^2 - xy + z^2 - 2xz + 6z$$

(7) Find the points on the sphere $x^2+y^2+z^2 = 4$ that are closest to and farthest from the point $(1, 1, 1)$. Do so using the method of Lagrange multipliers. (Hint: the function you wish to optimize is the distance between a given point (x, y, z) and the point $(1, 1, 1)$. However, optimizing the square of this distance is preferable. Why?) Avoid using a technical argument for determining which is closest and which is farthest and use some geometric reasoning/intuition instead.

(additional space to show work)