MULTIVARIABLE CALCULUS
EXAM 1
FALL 2025

Name:
Honor Code Statement:

Directions: Each question or one of its subparts is worth 5 points – you may omit any one of these by putting a slash through it. Justify all answers/solutions. No outside materials are permitted. A formula sheet is on the last page. The exam is proctored by permission of the Dean of the Faculty. Best of luck!

(1) [5 points] Evaluate this limit by making a change of variables to polar coordinates.

$$\lim_{(x,y)\to(0,0)} \frac{x^2+xy+y^2}{x^2+y^2}$$

Date: October 8, 2025.

(2) [5 points] Calculate the directional derivative of the function $f(x,y) = \frac{1}{x^2+y^2}$ at $\mathbf{a} = (3,-2)$ in the direction parallel to the vector $\mathbf{u} = \mathbf{i} - \mathbf{j}$.

(3) [5 points] Find the matrix of partial derivatives of the function $\mathbf{f}(x,y) = (x^2, xy, \sin(x))$. Use Theorem 3.10 to justify that the function is differentiable at the point $(1,1)$.

(4) [25 points]

(a) Find the equation of the plane that contains the following three points: $(1, 2, 3), (4, 0, 1)$ and $(1, 3, 0)$.

(b) Give a vector that is normal to the plane.

(c) Find the equation for a plane parallel to the one you found that contains the point $(0, 0, 0)$.

(d) Find the distance between the plane found in the first part and the point $(0, 0, 0)$.

(e) Find the parametric equations for the plane found in the first part of this question.

(5) [35 points] Consider the following function with codomain $\mathbb{R}$: $f(x, y) = 4x^2 + 9y^2$.

(a) State the domain and range of the function. Is the function onto? Why or why not?

(b) Provide a counterexample to the statement, "f is one-to-one."

(c) Determine/draw several level curves of the given function. Do so for the values $c = 0, c = 4, c = 9$ and one other value of c.

(d) Use this information to sketch or describe the graph of f.

(e) Find the gradient vector at the point $(1, 1)$.

(f) Use this information to find the normal line to the level curve C associated with the point $(1, 1)$ at this point. That is, find the line perpendicular to C.

(g) Find the equation of the tangent plane at the point $(1, 1, 13)$.

Change of coordinates

Cylindrical to Cartesian:

$$x = r\cos\theta,\ y = r\sin\theta, z = z$$

Cartesian to cylindrical:

$$r^2 = x^2 + y^2,\ \tan(\theta) = \frac{y}{x},\ z = z$$

Spherical to Cartesian:

$$x = \rho\sin\varphi\cos\theta,\ y = \rho\sin\varphi\sin\theta,\ z = \rho\cos\varphi$$

Cartesian to spherical:

$$\rho^2 = x^2 + y^2 + z^2,\ \tan(\varphi) = \sqrt{x^2 + y^2}/z,\ \tan(\theta) = \frac{y}{x}$$

Spherical to cylindrical:

$$r = \rho\sin(\varphi),\ \theta = \theta,\ z = \rho\cos(\varphi)$$

Cylindrical to spherical:

$$\rho^2 = r^2 + z^2,\ \tan(\varphi) = r/z,\ \theta = \theta$$