# MATH 746: Senior Seminar on Linear Algebra Methods for Combinatorial Problems Fall 2023

September 12, 2023

## Contents

# 1 Catalog Description

**MATH 0746 Senior Seminar on Linear Algebra Methods for Combinatorial Problems**
**Fall 2023**

A tutorial in the linear algebra methods for students who have completed work in Linear Algebra (and possibly Abstract Algebra) and at least one of Combinatorics, Graph Theory and Number Theory. We will study the linear algebra method and related theorems, along with applications to combinatorics, graph theory, number theory, and incidence geometry. Working independently and in small groups, students will gain experience reading advanced sources and communicating their insights in expository writing and oral presentations. Fulfills the capstone senior work requirement for the mathematics major.

# 2 Instructor

Instructor: John Schmitt
Office: Warner 205
E-mail: jschmitt@middlebury.edu
My webpage: http://community.middlebury.edu/∼jschmitt/
Course webpage: http://community.middlebury.edu/∼jschmitt/745.html
Office Hours: Tuesday 1pm–2:15pm, Thursday 10am–11am, Friday 2:15pm–3:45pm and by arrangement
Course meeting time and location: Tuesday and Thursday, 11:15am–12:30pm in Warner Hall 010

# 3 Purpose of the Seminar

The Senior Seminar in Mathematics has a primary purpose:

To give majors the opportunity to explore in depth a topic in pure or applied mathematics. This experience emphasizes some aspects of education not normally stressed in our regular courses: independent study, library research, organizing and internalizing a chunk of mathematics, expository writing, and verbal presentation of material.

# 4 Schedule

Thursday, September 21 (Week 3) – Have read some past theses.
Thursday, October 12 (Week 5) – Topic proposal is due for approval
Thursday, October 19 (Week 6) – Five-minute talk
Thursday, November 2 (Week 8)– Outline and bibliography due
Thursday, November 30 – Draft of paper due
Tuesday, December 5 and Thursday, December 7 – 20-minute talks (plus 5 minutes for questions)

Thursday, December 14 – Final paper (by 5pm)

Late Penalties: The department's policy normally requires a minimum penalty of a drop of one grade for each day, or portion thereof, the thesis is late.

You should begin with the realization that the senior seminar in mathematics is supposed to be the capstone of your educational experience at Middlebury. It should be your number one priority for the term. In the second half of the semester, large blocks of your time will be devoted to writing your paper.

This schedule requires the student to make an early commitment to a particular topic and to begin work on that subject as soon as possible. During the first few weeks of lectures, I will propose various possibilities for a topic. You might choose one of these or not. Regardless, your topic must be approved by me. You can eliminate some of the last minute all-nighters and panicky final days by disciplined work in the first half of the term. Set a definite period of time each day which you will devote to your thesis.

Students sometimes experience difficulty in their first independent learning experience. The structure imposed on you in regular courses – classes meeting two or three times a week, daily homework, scheduled examinations – makes it somewhat easier for you to organize your time and to discipline yourself to meet deadlines. Some of that structure is (deliberately) missing from the Senior Seminar. You will have to take more initiative in organizing your schedule to complete the work on your thesis.

# 5 Plagiarism

The habit of intellectual honesty is essential to both intellectual and moral growth. Effective evaluation of student work and helpful instruction can take place only in an environment where intellectual honesty is respected.

The relevant *Middlebury College Handbook* language is as follows:

"As an academic community devoted to the life of the mind, Middlebury College requires of every student complete intellectual honesty in the preparation of all assigned academic work.."

"Plagiarism is a violation of intellectual honesty. Plagiarism is passing off another person's work as one's own. It is taking and presenting as one's own the ideas, research, writings, creations, or inventions of another. It makes no difference whether the source is a student or a professional in some field. For example, in written work, whenever as much as a sentence or key phrase is taken from the work of another without specific citation of the source, the issue of plagiarism arises.

"Paraphrasing is the close restatement of another's idea using approximately the language of the original. Paraphrasing without acknowledgment of authorship is also plagiarism and is as serious a violation as an unacknowledged quotation...

"The individual student is responsible for ensuring that his or her work does not involve plagiarism. Ignorance of the nature of plagiarism or of College rules

may not be offered as a mitigating circumstance."

The use of generative artificial intelligence is prohibited.

As many theses in our department involve restatements of known theorems and the proofs of the same, the question of plagiarism may be relevant to you. I will be able to answer any questions you may have about this subject. It is your responsibility to consult with me.

# 6 Texts

Most of our shared reading will come from the following two sources.

*Thirty-three Miniatures: Mathematical and Algorithmic Applications of Linear Algebra* by Jiří Matoušek. An on-line preliminary version from the author and with the permission of the American Mathematical Society is available at `https://kam.mff.cuni.cz/~matousek/stml-53-matousek-1.pdf`. A listing of misprints for the text is also available at `https://kam.mff.cuni.cz/~matousek/la-ams.html`. (This embedded link doesn't want to work? Try a copy-paste.)

The unpublished *Linear Algebra Methods in Combinatorics with Applications to Geometry and Computer Science* by László Babai and Péter Frankl. A copy is available from one of the author's webpage at `https://people.cs.uchicago.edu/~laci/CLASS/HANDOUTS-COMB/BaFrNew.pdf`. (This embedded link doesn't want to work? Try a copy-paste.)

I recommend *On Writing Well*, 6th Ed., by William Zinsser as a good general guide to expository writing and the always indispensable *The Elements of Style* by William Strunk and E. B. White.

There are also several excellent short books on the writing of mathematics. These include:

- *How To Write Mathematics* by Norman Steenrod, Paul Halmos, Menahem Schiffer, and Jean Dieudonné,
- *A Primer of Mathematical Writing* by Steven G. Krantz,
- *Mathematical Writing*, edited by by Donald E. Knuth, Tracy Larrabee, and Paul M. Roberts, and
- *Handbook of Writing for the Mathematical Sciences* by Nicholas J. Higham.

A copy of these books are in the Davis Family Library.

# 7 Library Assistance

The College's reference librarians can be of enormous assistance to you in assisting you to identify and locate relevant materials whether in printed or digital format. You may wish to contact the science librarian most familiar with mathematics and computer science materials.

You may also apply at the Circulation Desk at the Library for special senior thesis privileges, such as extended checkout periods, a carrell, or a locker.

# 8 Thesis Expectations

Theses that earn higher grades have some of the following qualities in common:

1. *Well presented.* The thesis should have a minimum of typographical errors and misspelled words and be neat and evenly spaced. Take the time to run a spell checker and proofread. Spelling and grammar do count. Have a friend proofread it for you; new eyes will see mistakes you've missed. Even if the content is great, you won't get a top grade unless it is also presented in a readable, comprehensible manner.

2. *Relatively difficult mathematics.* You must learn something new, something you did not see in a class, and something with substance. The work does not have to be very broad, but it should be somewhat deep. One option is to take a narrow topic and learn a lot about it. You should understand the topic and be able to explain it in your own words.

3. *Independent work.* You should do most of the learning on your own. Read the material; try to work through proofs by yourself. Feel free to ask questions of me, but do not expect your me to present the material to you or to do the proofs for you. If you get stuck, it is perfectly acceptable to ask for hints or for you and me to work through a problem together. Ideally you should combine material from several sources and draw your own conclusions or arrange the material in an original way.

   For example, you might pose a problem. Then you could try to work out your own solution (or examples) and/or find several different solutions (or examples) from several different sources. You might discuss the similarities and differences of these solutions: Do they use different types of mathematics? Can you draw your own illustrations or examples of these methods? Insert some of yourself into your thesis – your opinions, your arrangement of material, your own proofs, or your examples.

4. *Correctness (or accountability).* Check your definitions; be sure that your theorems are correct and that your proofs make sense to you. Can you explain them in your own words? Be very careful that you're not just mimicking someone else's proof and that you really do understand the words you are using.

   There is a fine line here between doing your own work and plagiarizing someone else's. If you copy something word for word without using quotation marks (or setting it off in a narrow paragraph) and including a citation, that is plagiarism. Using well established definitions from the literature is not plagiarism, but you must acknowledge your source. It is not acceptable to string together paragraph after paragraph of quoted material. You should be doing most of the writing yourself, using quotations to support a point.

   For the expository parts of your thesis, gather the information and then express the ideas in your own words. The definitions can be quoted. The proofs should be, as much as possible, your own. That doesn't mean that you have to

prove everything yourself. Working through someone else's proof is a perfectly acceptable thing to do. However, when it comes time to write up your thesis, you should try, as much as possible, to express the ideas in your own words.

5. *Parts completed on schedule.* This means not only meeting all intermediate deadlines, but also holding regular (usually weekly) meetings with your adviser. This is not meant to be a last minute, night before project. This is meant to be a semester long research project. Get each part done on time! Get regular feedback from your me. You will need the time available to absorb and understand the difficult concepts. It you understand all of the ideas the first time through, then your problem (project, topic) may not be hard enough.

6. *Appropriate depth and length* Your thesis should delve deeply enough into the subject area that your analysis requires reasonably sophisticated undergraduate mathematics. Your treatment of the material should be sufficiently extensive to explore the topic thoroughly and carefully, but do not overdo the length. I anticipate your thesis to be between 20 and 25 pages in length. You will need to get permission to go beyond 25 pages. Your goal should be **Quality**, not **Quantity**.

7. *Some further suggestions* Given that we will examine algebraic arguments for combinatorial problems and that these are claimed to provide short, elegant proofs, it will be good if you can explain why a combinatorial argument is missing or somehow inadequate. Also, examples are useful to give and help ease the reader into the more demanding parts of the thesis; this is often a place where a student's individual fingerprint on the topic can begin to develop. Later it will develop by way of giving more details in proofs that are too short for an undergraduate reader that were found in the literature or by expositing on the history of the problem/theorem/etc.

# 9 Basis for Evaluation

In many departments of the College, a senior thesis is an option available for students seeking Honors in the majors. In our program, all majors are required to complete a one-term senior thesis. The distribution of grades in MATH 704 has followed a pattern quite similar to the distribution of grades in other senior level courses in the department. For your information, here is a distribution of MATH 704 grades from 1975 through Spring 2016 (note that A+ is no longer an available grade):

| Grade | Number | Grade | Number |
|---|---|---|---|
| A+ | 12 | C+ | 39 |
| A | 108 | C | 19 |
| A- | 141 | C- | 9 |
| B+ | 127 | D | 10 |
| B | 96 | F | 2 |
| B- | 62 | | |

Keep in mind that each year the department awards the Dr. Francis D.

Parker, '39 Mathematics Prize. The award was established in 1993 by Dr. Francis D. Parker, class of 1939. It is awarded to the graduating student for the best senior work in the mathematics department and is largely based upon the senior thesis, though other accomplishments (Putnam scores, Green Chicken scores, conference participation, etc.) are considered as well.

# 10 Typsetting Your Thesis

The Mathematics Department requires that all seniors compose their theses using TEX. TEX is a typesetting system created initially in the late 1970s by Stanford mathematician and computer scientist Donald Knuth. Knuth designed TEX to allow any individual to produce high-quality typeset books and articles using a reasonable amount of effort, and to provide a system that would give the exact same results on all computers, now and in the future.

TEX is generally considered to be the best way to typeset complex mathematical formulas, but, especially in the form of LATEX and other template packages, is now also being used for many other typesetting tasks. LATEX offers programmable desktop publishing features and extensive facilities for automating most aspects of typesetting and desktop publishing, including numbering and cross-referencing, tables and figures, page layout, bibliographies, and much more. LATEX was originally written in 1984 by Leslie Lamport and has become the dominant method for using TEX . Few people write in plain TEX anymore.

Software packages that incorporate LATEX and TEX are widely available on the web and are free of charge. We recommend MikTeX for Windows users and TeXShop for Macintosh users, though Overleaf is now perhaps dominant for writing LATEX .

Several books on using LATEX are available at the library. They are:

- *Learning LATEX* by David Griffiths and Desmond Higham.
- *A Guide to LATEX: Document Preparation for Beginners and Advanced Users* by Helmut Kopka and Patrick Daly.
- *The LATEX Companion* by Michel Gossens, Frank Mittelbach, and Alexander Samarin.
- *First steps in LATEX* by George Grätzer.
- *LATEX : a document preparation system: user's guide and reference manual* by Leslie Lamport.