Quantum Fluctuations:
From Nanotechnology to the Cosmos

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A Musical Note

Playing a musical note creates regular vibrations in the air, which are detected by our auditory systems. Two questions:

1. At what time does the note happen?
2. What is its pitch? (In physics terms: What is the frequency of the vibrations, in cycles per second?)

I claim that neither question has a definite answer:

1. The note extends over a range of times — we could talk about when we reach the start of the note, or the middle, or the end, but there's no unique time for the note.
2. This case is more subtle, but actually the pitch has a completely analogous ambiguity. The shorter the time for which the note is played, the wider the range of pitches are involved.

Musical Uncertainty

If we play the note for a short time, there's a narrow range of times involved. So the ambiguity in the time at which the note is played is small. But this is precisely when the ambiguity about the pitch is large — as you can hear by playing shorter and shorter samples of the same note.

Conversely, if we play the note for a long time, there's a wide range of times involved. So the ambiguity in the time at which the note is played is large. But this is precisely when the ambiguity about the pitch is small.

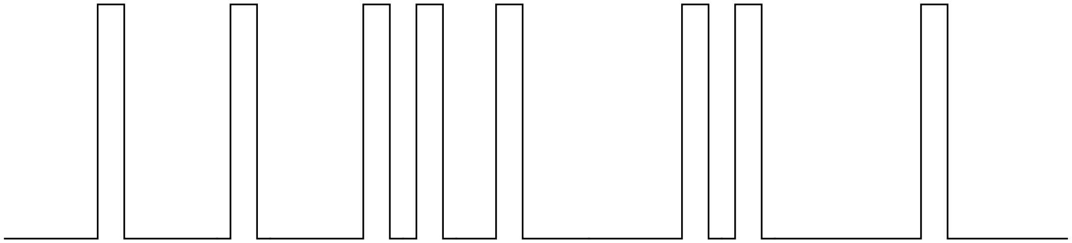
This uncertainty can be made mathematically precise:

$$\text{Uncertainty in Time} \times \text{Uncertainty in Pitch} \geq \frac{1}{4\pi}$$

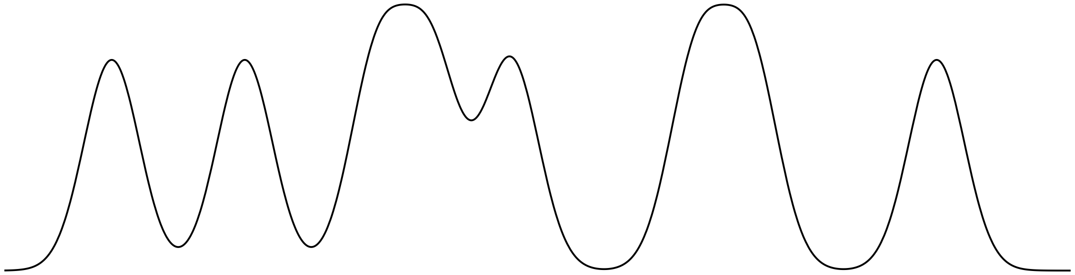
Internet Uncertainty

We said that a short pulse contains a wide range of frequencies.

This is why high speed internet (lots of short pulses) requires high bandwidth:



Low bandwidth version:



Low bandwidth of ordinary telephone is why it's hard to hear the difference between "b" and "v."

Heisenberg's Uncertainty Principle

In quantum mechanics, the motion of a particle in space works the exact same way.

1. Location of the particle $\Leftrightarrow$ Time of the note
2. Movement of the particle (momentum) $\Leftrightarrow$ Pitch of the note

and so ($h =$ Planck's constant):

$$\text{Uncertainty in Position} \times \text{Uncertainty in Momentum} \geq \frac{h}{4\pi}$$

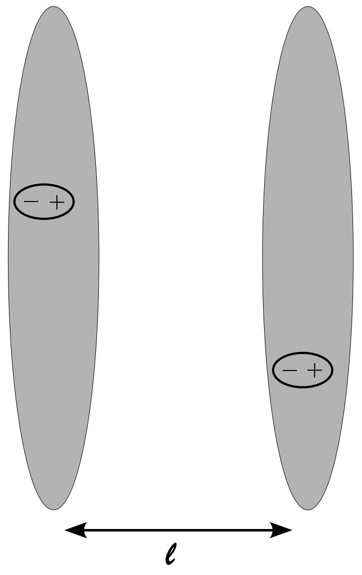
In quantum mechanics, position and momentum are not independent: They are tied together by the relationship between time and frequency.

Uncertainty and its Consequences

On the scale of everyday life, these uncertainties are very small. But for distances at the scale of atoms or semiconductors, they become important.

In particular, if we are uncertain about a particle's motion, we can't be sure it's at rest. Nothing can stand perfectly still!

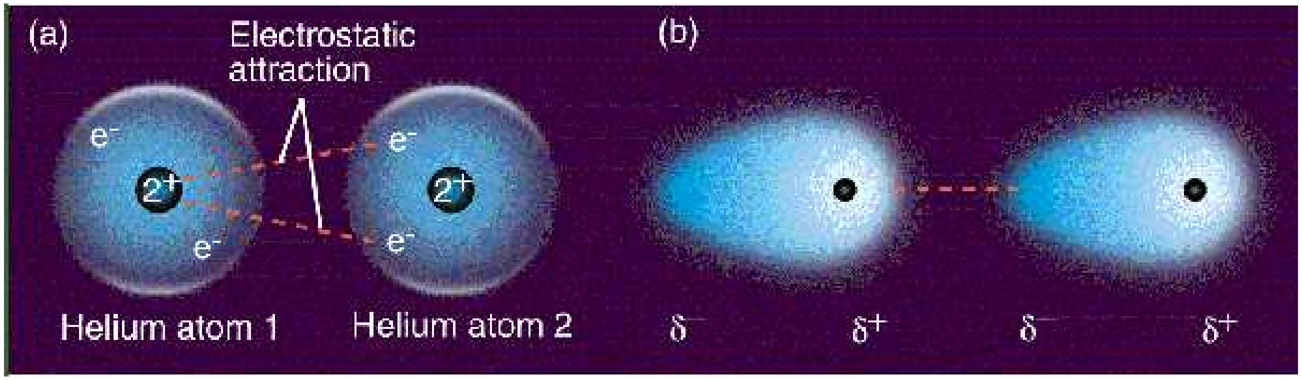
So, for example, electrons in a material have to move around. That creates regions with negative charge, and leaves behind regions with positive charge.



But then the opposite charges attract, and try to get closer together. But once they're closer together, their attraction is stronger than the repulsion of the like charges, creating an attractive force!

A Force From Nothing

In chemistry, these fluctuations appear as van Der Waals (more precisely, London dispersion) forces:



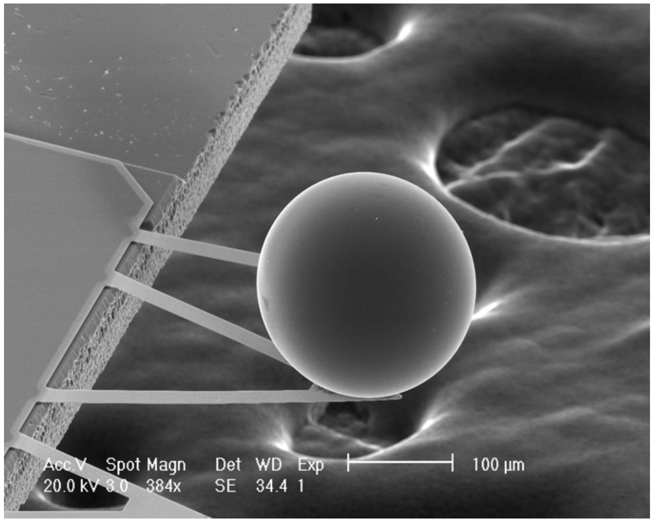
[Image from P. J. Brucat, General Chemistry I, University of Florida]

[Image from Wikipedia]

Calculating Fluctuation Forces

In 1948, Hendrik Casimir predicted a quantum-mechanical force between uncharged conducting parallel planes. The first precision verifications did not take place until 1994, however.

Lamoreaux; Mohideen and Roy;
Bressi, Carugno, Onofrio, and Ruoso;
Chan, Aksyuk, Kleiman, Bishop, and Capasso



[Mohideen & Roy 1998]

Until recently, theoretical calculations relied almost exclusively on Casimir's result, approximating everything as parallel planes.

Our goal: Develop a general, systematic, practical method for calculating Casimir forces, appropriate for edges and other surfaces with high curvature.

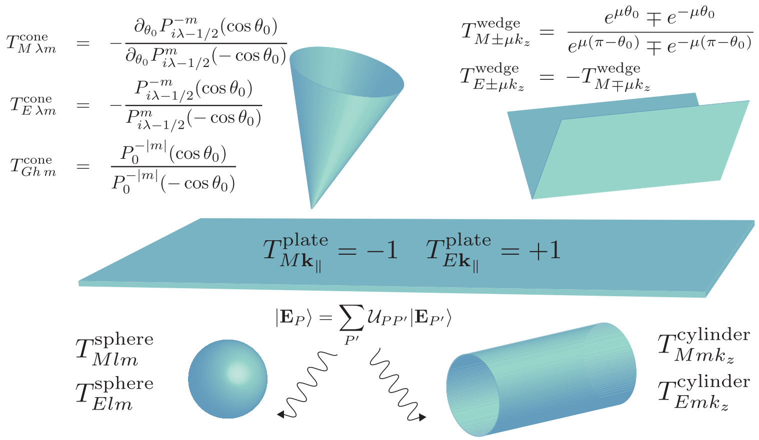
Calculational Tools

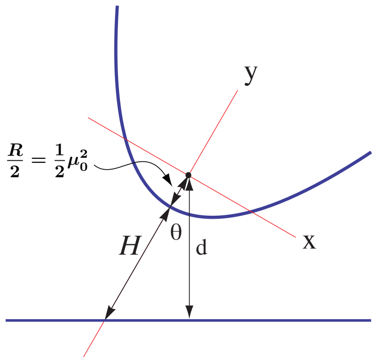
Our group has worked on new tools to calculate Casimir forces for a wider range of situations. The key idea: Start from information about how electromagnetic waves (infrared, visible light, ultraviolet) reflect (classically) from each object individually.

[Emig, Graham, Jaffe, Kardar, Rahi]

We use waves appropriate to the objects' shapes (planar, cylindrical, spherical, etc.) and then weave together the reflection data using geometrical formulae relating one kind of wave to another. This step encodes the objects' relative positions and orientations.

This approach enables us to calculate Casimir forces in a wide variety of different situations:





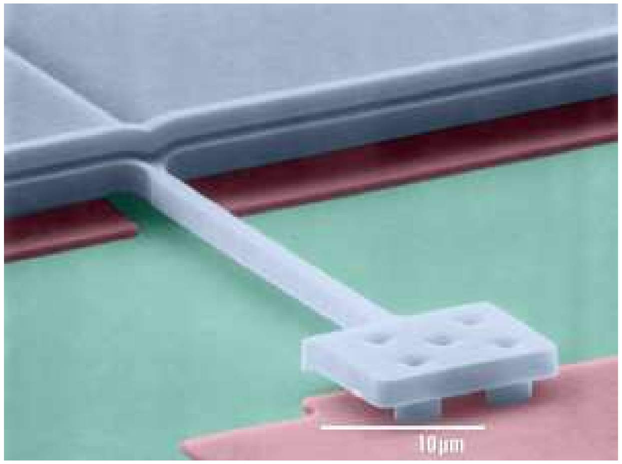
Applications

Micromechanical devices:

- accelerometers
- piezoelectrics
- pressure sensors
- gyroscopes

are already incorporated in a wide range of technological applications.

[Images from Canadian Assoc. of Road Safety Professionals, Wikipedia]



[Chan 2007]

As these technologies move to smaller and smaller length scales, Casimir forces and other quantum-mechanical effects will become important (for better or worse).

Deviations from the predicted Casimir forces could also allow us to discover new physical laws!

On to the Cosmos

The "quantum fluctuations" that give rise to the Casimir force are also the source of a major unsolved mystery in cosmology.

Because of the uncertainty principle, empty space can't truly be empty — that would again imply too much certainty. For example, the electric and magnetic fields can't be exactly zero — there's an uncertainty between them as well. But if there are fluctuations in empty space, then there should also be energy in empty space.

Energy in empty space doesn't affect most physical processes, since we can't extract it; it's just always there.

But according to $E = mc^2$, a source of energy is also a source of mass. So gravity should notice this energy. And it does so in a surprising way.

Dark Energy

Ordinary matter slows the expansion of the universe, since it pulls matter back together. But the energy of empty space, known as dark energy, has the opposite effect: It accelerates this expansion. The key difference is the pressure it produces.

General relativity says that both energy ($= mc^2$) and pressure contribute to the force of gravity. But pressure is three times more important than energy in three-dimensional space.

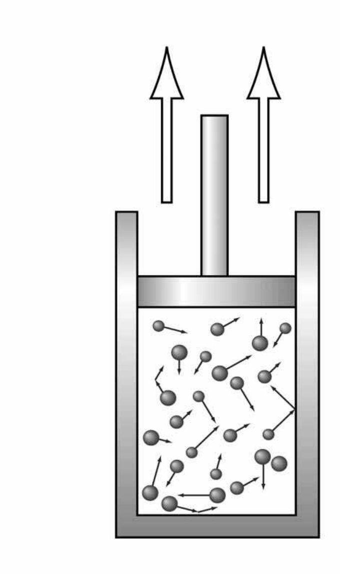
The pressure of ordinary matter is very small on cosmological scales — atoms and molecules need to be moving close to the speed of light for their pressure to be important.

Under (Negative) Pressure

If you let a gas of ordinary matter expand, its energy goes down: It does work.

The same result holds when the expansion is caused by the expansion of the universe instead of a piston.

[Image from WikiPremed]



Dark energy gives a fixed energy per unit volume to empty space. So as the universe expands, the volume increases, and the energy goes up! (And that's where we think all the energy in the universe originally came from. . . .)

So it's a form of energy with negative pressure. That turns out to reverse its effect on the expansion of the universe: Rather than slow the expansion down, dark energy speeds it up!

Dark Energy

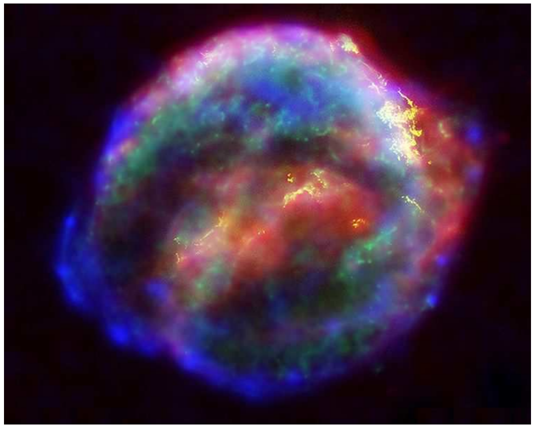
Einstein was the first to identify the possibility of dark energy, when he developed the theory of general relativity. But when general relativity turned out to work just fine without dark energy, he rejected the possibility, calling it his "greatest blunder."

Later, when the inevitability of quantum fluctuations was understood, the mystery returned: The acceleration caused by quantum fluctuations should be more than big enough to blow the universe apart before stars, galaxies, etc. could form. So physicists assumed there must be (but couldn't find) some law that prevented quantum fluctuations from contributing to gravity.

But in 1998 a new experiment made the situation even more confusing. . . .

Supernova Redshifts

Supernovae are cataclysmic explosions of dying stars. They are so bright that we can observe them in distant galaxies, and they are "standard candles" in that they have a predictable intrinsic brightness. Comparing this intrinsic brightness to a supernova's apparent brightness determines its distance, and looking at its redshift determines how fast it is moving away from us.



[Image from Wikipedia]

Seeing supernovae that are so far away enables us to look back in time at earlier stages in the universe's expansion. Two groups studying distant supernovae found remarkable result: The redshift is too small for galaxies in the past compared to today. So the expansion of the universe has accelerated, driven by dark energy, which fills empty space.

Quantum Mechanics to the Rescue?

The energy density we observe in dark energy is still very small in comparison to what we'd expect from quantum fluctuations. Even in the most optimistic scenario it's still off by a factor of 10^{15}.

We only notice dark energy because the universe is so big (and mostly empty). But it makes up about 73% of the energy in the universe. Which yields another mystery: The amount of energy in other forms of matter is roughly constant in time, while the amount of energy in dark energy grows as the universe expands.

So earlier in the history of the universe, the fraction in dark energy was 0.001%; later in the history of the universe, it will be 99.999%.

We live in the special time where these numbers are even close to 50%. Just a coincidence?

An Explanation?

A recent but controversial explanation is the anthropic principle.

Our current cosmological models can naturally incorporate multiple universes, each with their own physical laws. Those universes that have a more “normal” amount of dark energy would get blown apart before stars or galaxies could form. So we live in the rare universe in which we could exist to ask questions about our universe.

It’s also the case that small changes in other fundamental constants would make life as we know it impossible (for example, by making nuclei of common elements unstable).

But maybe this is just a way of saying that we give up...