

Please note that some browsers require the animation to be clicked first to activate it.

Enter the operation and near-number arguments here (see below);  
press  $\rightarrow$  to display the result

The interface displays three horizontal number lines illustrating the operation  $0^+ \times +\infty$ . The top number line ranges from -2 to 2 with a red arrow pointing right from 0. The middle number line ranges from 0 to 10 with a red arrow pointing left from 5. The bottom number line ranges from -20 to 20 with a red arrow pointing right from 0. A magnifying glass icon is positioned below the bottom number line. In the top right corner, there are three buttons: a left-pointing double arrow (labeled 'R'ewind), a right-pointing double arrow (labeled 'P'lay/pause), and a 'ZOOM' button.

Rewind, Play/pause, or Zoom the animation

View the graphical result

- Near-numbers are entered as usual (see instructions for the basic near-number viewer);  
Press  $\text{SPACE}$  to cycle through the locations.
- Arithmetical operations:  $\text{+}$   
 $\text{-}$   
 $\text{/}$  for  $\div$   
 $\text{x}$  or  $\text{*}$  for  $\times$

A few illustrative examples of near-number arithmetic computation are below. At some level, the text is irrelevant—you can tell what the result is simply by applying visualizing the picture (no memorization needed!).

We focus particularly on *indeterminate forms*, which are illuminated exceedingly well by the near-number approach.

### Addition and subtraction

$1^+ + 2^- \rightarrow 3^\star$ : This simple arithmetic is a warm-up for indeterminacy; depending on the numbers that we choose from each summand, we end up with sums below, above, and equal to 3 in our result. However, as the two summands pinch toward 1 and 2 (respectively), the results do pinch ever closer to 3; in summary, our result  $\rightarrow 3^\star$ .

$+\infty - 1000 \rightarrow +\infty$ : While not indeterminate, this computation shows us explicitly how  $+\infty$  can absorb any finite summand: at first the result takes negative values; however, as  $+\infty$  pushes farther to the left,  $-1000$  stays still, so that  $+\infty$  eventually wins the battle and our result clearly fits the form of  $+\infty$ .

Moreover, it is clear that the only difference if we'd chosen an even larger negative number is that it would have taken a bit longer for the  $+\infty$  to win—but the result would be the same.

$+\infty - +\infty \rightarrow \star$ : The first classical case of indeterminacy: do the two  $+\infty$ 's cancel each other? No—first of all, because cancellation is for numbers, not near-numbers. But we can see just what *is* happening if care to watch the animation:

No matter how long we watch, both terms will contain larger and larger numbers; their difference will be the set of all possible differences that we can form by choosing one number in each term. If we fix a number in the first term and choose larger and larger numbers in the second term, we obtain values that are larger and larger negative numbers (and don't stop). Conversely, if we fix a number in the second term and choose larger and larger numbers in the first, we obtain values that are larger and larger positive numbers. What's more, we obtain all of the numbers in between, as well—so, however long we allow the near-numbers to pinch, the result is always the same: the set of all real numbers. This clearly doesn't fit into any basic finite or infinite near-number, so we simply say that this “indeterminate form”  $\rightarrow \star$ .

$+\infty + +\infty \rightarrow +\infty$ : In contrast to the case above, there is nothing the least bit indeterminate about this sum; as the first term pushes off to the right and the second term does likewise, their sum pushes farther and farther to the right—exactly what is demanded to  $\rightarrow +\infty$ .

Technically, near-number product and quotient computations can be performed in terms of near-number addition and subtraction via logarithms—but it is good practice to think through some more examples from scratch.

### Multiplication and division

$+\infty \div 10000 \rightarrow +\infty$ : Dividing the numbers in each slice of  $+\infty$  shrinks them by a finite factor—but as  $+\infty$  pushes ever farther rightward, these squeezed results follow suit, pushing ever farther rightward to give a result that  $\rightarrow +\infty$ .

$0 \times +\infty \rightarrow 0$ : Zero times anything is, indeed, zero; no matter how far the numbers in  $+\infty$  push to the right, multiplication by zero results simply in zero. . .

$0^+ \times +\infty \rightarrow \star$ :  $\dots 0^+$ , on the other hand, is a different story. Choosing any number from the first factor and taking larger and larger numbers from the second factor, we find ever larger numbers in the product. Conversely, choosing any number from the second factor and taking smaller and smaller numbers from the first, we find ever smaller numbers. In fact, the product picks up all numbers between, and no matter how far in the animation we go, the product is exactly the same: all positive numbers. Because this result doesn't fit into any basic near-number, we simply say that this product  $\rightarrow \star$ .

$0^- \times 0^- \rightarrow 0^+$ : Here, as each factor squeezes toward zero (on the negative side), the products squeeze toward zero, as well—however, being products of negative numbers with negative numbers, they'll squeeze in from the positive side, thus the result  $\rightarrow 0^+$ .

$0^+ \div 0^+ \rightarrow \star$ : In any given slice, we see small positive numbers in each term. As usual, we form the quotient by dividing all numbers in the first term by all numbers in the second term. Choosing one number from the top term and dividing it by smaller and smaller numbers from the bottom term results in ever larger numbers in the product; similarly, choosing one number from the bottom term and smaller and smaller numbers from the top term, we find ever smaller positive numbers in the product. Thus, independent of how far the terms pinch, the result is always the same: all positive numbers—this fits into no basic near-number, so we simply say that the quotient  $\rightarrow \star$ .