1. Motivating Probabilism

Reliabilists hold that epistemic justification is a function of the *indefinite* probabilities of our beliefs.

- *Indefinite* probabilities attach to general classes of properties.

Probabilists hold that epistemic justification is a function of the *definite* probabilities of our beliefs.

- *Definite* probabilities attach to specific instances.

But what exactly *is* probability?

- *Frequentists* assert that probabilities are ratios between classes of events.
- *Subjectivists (Bayesians)* assert that probabilities are *degrees* of belief.
- Reliabilists are frequentists; probabilists *tend* to be subjectivists. We will bracket objective probabilism in what follows.

1.1. Comparison with deductive logic

Reminders:

- An inference is *deductively valid* iff$_{\text{df}}$ it is *impossible* that its premises are true and its conclusion is false.
    - Ex. *Modus ponens*: If *p* then *q*, *p*. Therefore, *q*.
- An inference is *inductively strong* iff$_{\text{df}}$ it is *improbable* that its premises are true and its conclusion is false.

Deductive logic rationally constrains our beliefs in two ways:

- *Synchronically*, deductive logic requires a set of beliefs to be *consistent;*
    - Inconsistent beliefs are *deductively incoherent*
- *Diachronically*, deductive logic constrains admissible *changes in belief* through *rules of inference*, e.g., *modus ponens*.

Similarly, Bayesians suggest that there are rules of *inductive* logic that place analogous constraints on *degrees of belief*. Specifically, the laws of probability theory provide synchronic criteria of *probabilistic coherence* and diachronic rules of probabilistic inference.

1.2. What are the laws of probability theory?

The probability calculus:

- *Axiom 1:* $0 \leq \text{prob}(p) \leq 1$.
- *Axiom 2:* If *p* and *q* are logically incompatible with each other, then prob(*p* or *q*) = prob(*p*) + prob(*q*).
- *Axiom 3:* If *p* is a tautology (i.e. a logical truth), then prob(*p*) = 1.

1.2.1. What is Conditionalization?

For Bayesians, the most important rule of probabilistic inference is called *Conditionalization:*

- *1st Stage*: *S* begins with initial or *prior* probabilities $prob_i$.
- *2nd Stage:* One acquires new evidence. Since this comes after the prior probabilities, it is a *posterior* or final probability. If we acquire evidence *e*, we are certain of *e*, i.e. $prob_f(e) = 1$
    - *e* is assumed to state the totality of one's new evidence and to have initial probability greater than zero.
- One systematically transforms one's initial probabilities to generate final or *posterior* probabilities $prob_f$ by *conditionalizing* on *e*.

For any statement *h:*

$$prob_f(h|e) = \frac{prob_i(e|h) * prob_i(h)}{prob_i(e)}$$

Here $prob_i(e|h)$ is called the *likelihood* (of *e* on *h*), and $prob_i(e)$ is called the *expectedness* (of *e*).

Consider some of the intuitive aspects of this mathematical formula.

- *The higher the likelihood, the better confirmed the hypothesis.* If you had high level of confidence that certain pieces of evidence would obtain if a particular hypothesis were true at Stage 1, then finding that evidence at Stage 2 certainly counts in favor of the hypothesis.

- *The higher the prior in the hypothesis*, *the better confirmed the hypothesis*. This is a kind of epistemic conservatism. If you've got a lot of confidence in a hypothesis at Stage 1, then most evidence at Stage 2 will convince you that the hypothesis continues to be a good one. (Conversely, it would take fairly astounding counterevidence at Stage 2 to convince you to revise your hypothesis).
- *The lower the prior in the evidence, the better confirmed the hypothesis*. A hypothesis that makes bold predictions gains credence when those bold predictions pan out.

2. Bayesian epistemology

A person is justified in believing *p* iff the subjective probability of *p* is sufficiently high. (*The Simple Rule*)

Two interpretations of subjective probability:

Descriptive subjectivism: Probability = a person's *actual* degree of belief in a proposition.

Normative subjectivism: Probability = a person's *rational* degree of belief given his overall situation.

2.1. *Problem with descriptive subjectivism*

1. If descriptive subjectivism is true, then people's degrees of belief conform to the probability calculus.
2. <u>People's degrees of belief do not conform to the probability calculus.</u>
3. ∴ Descriptive subjectivism is false. (1, 2)

2.2. *Dutch book argument for normative subjectivism*

DB1. If your beliefs do not conform to the probability calculus, then one can always make a set of bets against you that guarantee a loss for you.

DB2. If one can always make a set of bets against you that guarantee a loss for you, then you are <u>irrational.</u>

DB3. ∴ If your beliefs do not conform to the probability calculus, then you are irrational. (DB1, <u>DB2)</u>

DB4. ∴ If you are rational, then your beliefs conform to the probability calculus. (DB3)

A "Dutch book" is a set of bets that guarantee a loss for the bettor.

2.2.1. *Objections to Dutch book argument*

- DB2 conflates "practical" rationality (adopting the means that will get you what you want) versus "epistemic" rationality (adopting the means that will get you to believe only truths).
- Even if we grant that DB2 is referring to epistemic rationality, it may be the case that: (a) your beliefs violate the probability calculus (so the Dutch book can be run against you), (b) you have no reason to think that your beliefs violate the probability calculus, and (c) you are epistemically rational.
- There is no unique degree of belief one should have: given a set of beliefs $b_1, \ldots, b_n$ that violate the probability calculus, one could revise b_1 OR revise b_2 … OR revise b_n (or combinations thereof) to get the overall set to conform to the probability calculus. However, this would mean that there are many rational/justified degrees of beliefs that two people can have when given the same evidence.
- (Not in P&C): since degrees of beliefs are internal states, subjective probability is not externalist-friendly. Though perhaps we cannot *access* these probabilities?

3. Probabilism Problematized

3.1. *Tautologies*

1. *Simple Probabilism* is true iff_{df} a person is justified in believing *p* iff the probability of *p* is sufficiently high.
2. $0 \leq \text{prob}(p) \leq 1$ (axiom of the probability calculus)
3. <u>If *p* is a tautology (i.e. a logical truth), then prob(*p*) = 1. (axiom of the probability calculus).</u>
4. ∴ If Simple Probabilism is true, then a person is always justified in believing logical truths.
5. <u>However, people are not always justified in believing logical truths.</u>
6. ∴ Simple Probabilism is false.

3.2. Arbitrary priors

What if someone has completely absurd prior probabilities (e.g., P_i (all emeralds are puppies) = 1; P_i(all emeralds are green) = 0)?

- One view, objective Bayesianism, holds that there must be rational constraints on prior probabilities.
- Another view, subjective Bayesianism, holds that there need not be such rational constraints.
  - This isn't so bad, since even subjects with very different priors will converge on common posteriors given a suitably long series of shared observations.

3.3. *Old evidence*

Recall that one of the virtues of Bayesian confirmation theory is that it has the intuitive consequence of placing high value on bold predictions, i.e., on hypotheses that predict evidence with low prior probability. However, this cuts both ways—Bayesianism places a low value on more mundane predictions and explanations in which the prior probability of the evidence is quite high: as in the case of old evidence.

3.4. *New theories*

The simple introduction of a new alternative hypothesis is sufficient for eroding support for a well-entrenched theory.