Sum Degree of No Class

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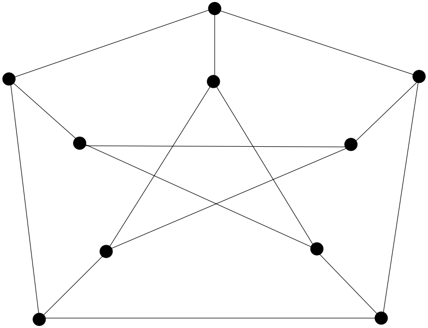
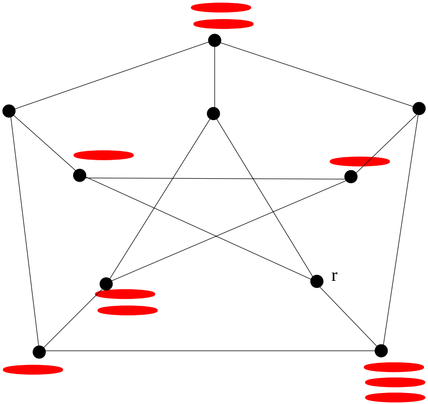
May 2007
joint work with Anna Blasiak

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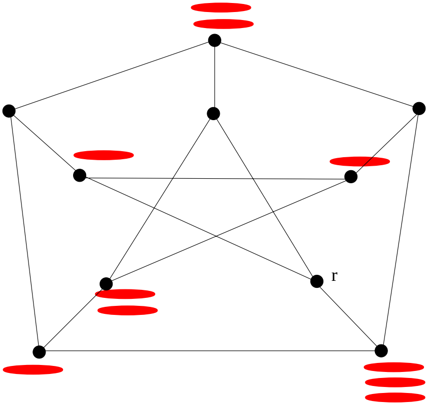
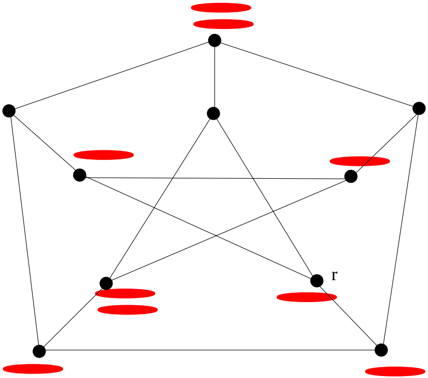


A *pebbling move* consists of removing two pebbles from a vertex and placing one of them on an adjacent vertex.



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Pebbling number of a graph G, denoted $\pi(G)$, is the least number of pebbles necessary to guarantee that regardless of distribution of pebbles and regardless of target vertex there exists a sequence of pebbling moves that enables us to place a pebble on the target vertex.

Lower Bound on the Pebbling Number

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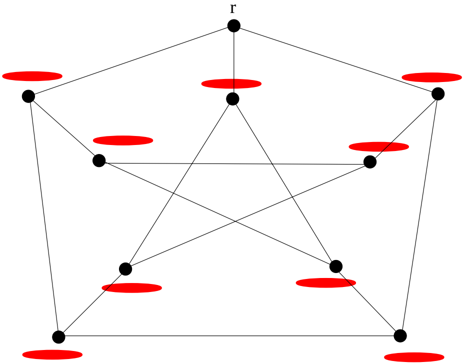

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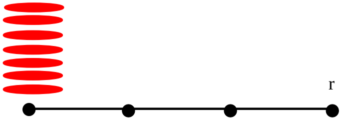

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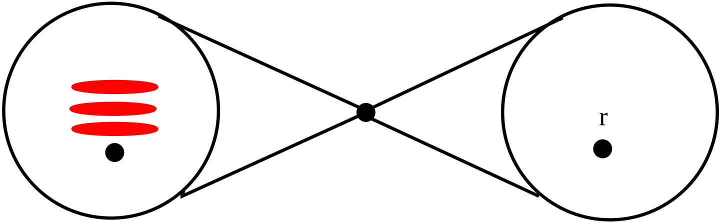
Class 0 graphs include:

- K_n
- $K_{s,t}, \quad 2 \leq s \leq t$
- Q_d (F. Chung, '92)
- Petersen graph
- Kneser graph (certain instances)

Problem:

Find necessary and sufficient conditions for G to be Class 0.

Class 0 graphs have connectivity at least two



Edge density for Class 0

Theorem (Pachter, Snevily, Voxman - '95) Let G be a connected graph with $n \geq 4$ vertices and $|E(G)|$ edges. If $|E(G)| \geq \binom{n-1}{2} + 2$, then G is Class 0.

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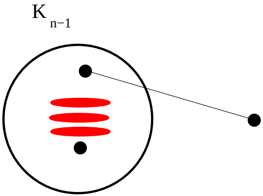
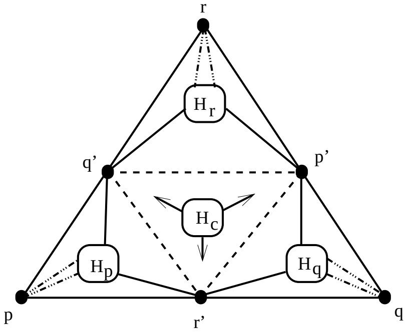


Theorem (Clarke, Hochberg, Hurlbert, '97) If a graph G has diameter two, connectivity at least two and $\pi(G) > n$, then $G \in \mathcal{F}$, where $\mathcal{F}$ is the following family of graphs.



(solid line = all edges present, dashed line = at least two edges exist, arrowed lines = at least two edges per vertex in H_c exist, double-dashed line = at least one edge per component)

A consequence of this result is:
Theorem (CHH, '97) If $diam(G) = 2$ and connectivity of G is at least three then G is Class 0.

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Another result:
Theorem (Czygrinow, Hurlbert, Kierstead, Trotter) There is a function $k(d)$ such that if G is a graph with connectivity at least $k(d)$ then G is Class 0. Moreover,

$$\frac{2^{diam(G)}}{diam(G)} \leq k(d) \leq 2^{2diam(G)+3}.$$

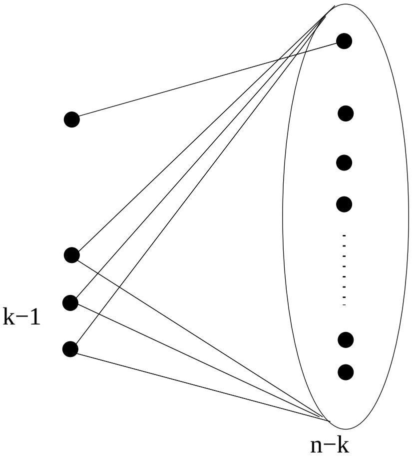
Degree Sum Result

Let $\sigma_k(G) =$
$min\{deg(x_1) + \ldots + deg(x_k) | x_1, \ldots, x_k$ are independent in $G\}$.

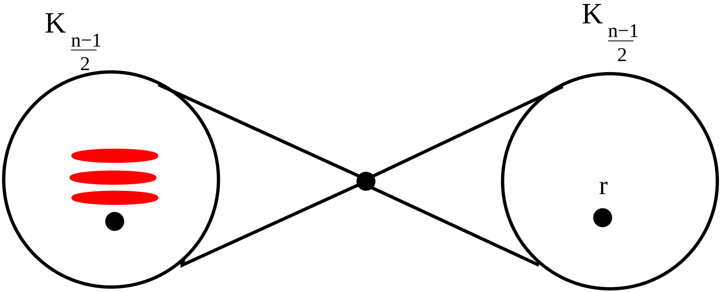
Theorem (Blasiak, S.) For $k = 2$ if $\sigma_k(G) \geq (k-1)(n-k) + 2$, then G is Class 0.

Corollary If $\delta(G) \geq \lceil \frac{n}{2} \rceil$ then G is Class 0.

Showing sharpness



Showing sharpness (for min degree)



Proof Sketch

Let G be a graph with $\sigma_k(G) \geq (k-1)(n-k)+2$ for $k=2$ and suppose $\pi(G) > n$.

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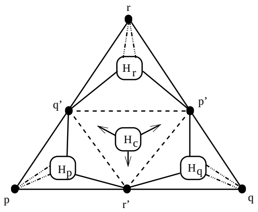

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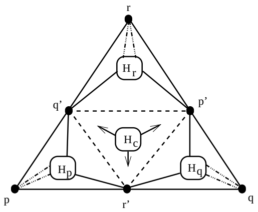

For $k=2$ we can find two pairwise non-adjacent vertices, namely p and q, with degree sum at most $(k-1)(n-k)+1$.

Open Problems and Questions

See the survey, "Recent Progress in Graph Pebbling" by G. Hurlbert.