Generalizing the degree sequence problem

John R. Schmitt
Middlebury College

The degree sequence problem, determining when there exists an n-vertex graph whose list of vertex degrees matches a given list of n non-negative integers, has been well-studied. One solution, provided by Erdős and Gallai, gave a set of n inequalities to check. They subsequently showed that in the case of a positive solution if the terms of the degree sequence summed to at least $2n-2$, then there exists a graph realizing the degree sequence which contains a spanning tree. In the early 90s, Erdős, Jacobson and Lehel (EJL) conjectured that if the terms of a degree sequence summed to at least $(t-2)(2n-t+1)+2$, then there exists a graph realizing this degree sequence containing a complete graph on t vertices. We discuss our proof to this conjecture. Also, in the hopes of proving a generalization of the EJL-conjecture, we discuss a generalization of the degree sequence problem as recently given by Amanatidis, Green and Mihail. Their generalized degree sequence problem is as follows: given a list of n non-negative integers *and* a square matrix whose entries are non-negative integers, determine if there is a graph whose list of vertex degrees matches the given list and the number of edges between the set of vertices of degree d_i and the set of vertices of degree d_j matches the (i, j)-entry of the given matrix.