LINEAR ALGEBRA
EXAM 3
SPRING 2017

Name:
Honor Code Statement:

Signature:

Directions: Complete all problems. Justify all answers/solutions. Calculators, cell-phones, texts, and notes are not permitted – the only permitted items to use are pens, pencils, rulers and erasers. Please turn off all electronic devices – in fact, you shouldn't have any with you. Additional blank white paper is available at the front of the room – you are not permitted to use any other paper. Good luck!

(1) [10 points] Apply the Gram-Schmidt Process to the following set of vectors in $\mathbb{R}^3$, $\mathbf{u_1} = (1, 1, 1), \mathbf{u_2} = (0, 1, 1), \mathbf{u_3} = (0, 0, 1)$. Then normalize the vectors you obtain.

Date: May 18, 2017.

(2) [5 points] Find the eigenvalues of the following matrix A. [5 points] Then find an basis for each eigenspace. [2 points] Use the eigenvalues to determine whether or not the matrix is invertible. [3 points] State why or why not the matrix is diagonalizable.

$$A= \begin{bmatrix} 1 & 3 \\ 4 & 2 \end{bmatrix}$$

(3) [15 points - 5 points for each part] Find (1) the orthogonal projection of $\mathbf{b}$ onto Col A, (2) a least-squares solution of $A\mathbf{x} = \mathbf{b}$, and (3) the least-squares error.

$$A= \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & -1 \\ 0 & 1 & 1 \\ -1 & 1 & -1 \end{bmatrix}$$

and $\mathbf{b}= \begin{bmatrix} 2 \\ 5 \\ 6 \\ 6 \end{bmatrix}$

(4) [3 points each] Each of the following statements is false. Amend each statement in as few words or symbols as possible, without insertion or deletion of the word "not" (that is, without negating the conclusion), to make a true statement or explain why it's false, which can be done by pointing to a theorem or giving a counter-example.

(a) The eigenvalues of a 2×2 matrix are on its main diagonal.

(b) An $n \times n$ matrix A is diagonalizable if A has n eigenvectors.

(c) For any $\mathbf{v} \in \mathbb{R}^3$ any scalar $c \in \mathbb{R}$, $\|c\mathbf{v}\| = c\|\mathbf{v}\|$.

(d) If a set $S = \{\mathbf{u_1}, \ldots, \mathbf{u_p}\}$ has the property that $\mathbf{u_i} \cdot \mathbf{u_j} = 0$ whenver $i \neq j$, then S is an orthonormal set.

(e) A row replacement operation on a matrix A does not change the eigenvalues.

(5) [5points] REMOVED FROM EXAM

(6) [10points] Define $T : \mathbb{P}_2 \to \mathbb{R}^3$ by $T(\mathbf{p}) = (\mathbf{p}(-1), \mathbf{p}(0), \mathbf{p}(1))$. Find the matrix for T relative to the basis $\{1, t, t^2\}$ for $\mathbb{P}_2$ and the standard basis for $\mathbb{R}^3$.

(7) [10 points] Prove the following theorem.

THEOREM If $S = \{\mathbf{u_1}, \ldots, \mathbf{u_p}\}$ is an orthogonal set of nonzero vectors in $\mathbb{R}^n$, then S is linearly independent.