1. [8 points] The following matrix is a block partitioned matrix. Show/prove that the matrix has an inverse AND determine this inverse based upon the block entries.

\[
\begin{bmatrix}
I_p & 0 \\
A & I_q
\end{bmatrix}
\]
2. [8 points] The following is an \( LU \)-factorization of \( A \). Use this factorization to solve the matrix equation \( Ax = b \), where \( b \) is given below.

\[
L = \begin{bmatrix}
1 & 0 & 0 \\
1/2 & 1 & 0 \\
3/2 & -5 & 1 \\
\end{bmatrix}
\]

\[
U = \begin{bmatrix}
2 & -2 & 4 \\
0 & -2 & -1 \\
0 & 0 & -6 \\
\end{bmatrix}
\]

\[
b = \begin{bmatrix}
0 \\
-5 \\
7 \\
\end{bmatrix}
\]
3. [8 points] Compute the determinant of the following matrix. Justify your steps.
\[
G = \begin{bmatrix}
4 & 8 & 8 & 8 & 5 \\
0 & 1 & 0 & 0 & 0 \\
6 & 8 & 8 & 8 & 7 \\
0 & 8 & 8 & 3 & 0 \\
0 & 8 & 2 & 0 & 0 \\
\end{bmatrix}
\]

[2 points] Is \( G \) invertible? Why?
4. [6 points] The following set of vectors is not a basis for $\mathbb{R}^3$. Find a basis for $\mathbb{R}^3$ that contains this set. Justify your answer.

\[ b_1 = \begin{bmatrix} 2 \\ 2 \\ 2 \end{bmatrix} \]

\[ b_2 = \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix} \]
5. [6 points] Let \( H = \left\{ \begin{bmatrix} x \\ y \end{bmatrix} : x^2 + y^2 = 0 \text{ or } x^2 + y^2 = 1 \right\} \). Is \( H \) a subspace of \( \mathbb{R}^2 \)? Why or why not?

6. [4 points] With regard to question 2, compute \( \det A \) without computing \( A \).
7. [6 points] Suppose a $5 \times 6$ matrix $A$ has four pivot columns.

(a) What is $\dim \text{Nul } A$? Why?

(b) Is $\text{Col } A = \mathbb{R}^4$? Why or why not?

8. [4 points] State the Spanning Set Theorem.
9. [8 points] Let $A = \{a_1, a_2, a_3\}$ and $B = \{b_1, b_2, b_3\}$ be bases for a vector space $V$, and suppose $a_1 = 4b_1 - b_2, a_2 = -b_1 + b_2 + b_3, a_3 = b_2 - 2b_3$.

(a) Find the change-of-coordinates matrix from $A$ to $B$.

(b) Use this to find $[x]_B$ for $x = 3a_1 + 4a_2 + a_3$. 

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10. [10 points] Prove the Unique Representation Theorem: Let \( B = \{b_1, b_2, \ldots, b_n\} \) be a basis for a vector space \( V \). Then for each \( x \) in \( V \), there exists a unique set of scalars \( c_1, c_2, \ldots, c_n \) such that \( x = c_1 b_1 + \cdots + c_n b_n \).