Linear Algebra
Exam 2 - Spring 2026

April 16, 2026

Name:
Honor Code Statement:

Directions: Complete all problems. Justify all answers/solutions. Calculators, cell-phones, texts, and notes are not permitted – the only permitted items to use are pens, pencils, rulers and erasers. Please turn off all electronic devices – in fact, you shouldn't have any with you. Additional blank white paper is available at the front of the room – you are not permitted to use any other paper. The exam is proctored with permission of the Dean of the Faculty. Good luck!

1. [8 points] Consider the following 2×2 matrix. Find a steady-state vector of this matrix M.

$$M = \begin{bmatrix} 0.75 & 0.1 \\ 0.25 & 0.9 \end{bmatrix}$$

2. [10 points] Compute the determinant of the following matrix. Then check your answer by taking a second approach to the computation. Is the matrix invertible?

$$A = \begin{bmatrix} 2 & 1 & 0 & 0 & 0 \\ 3 & 4 & 0 & 0 & 0 \\ 0 & 0 & 2 & 2 & 0 \\ 1 & 1 & 1 & 1 & 0 \\ 2 & 2 & 1 & 1 & 2 \end{bmatrix}$$

If a certain parallelepiped in $\mathbb{R}^5$ has volume 5, what is the volume of the parallelepiped that is the image of it under the transformation $\mathbf{x} \mapsto A\mathbf{x}$.

3. [6 points] Is the following set a subspace of $\mathbb{R}^2$? Justify your answer.

$$S = \{ \begin{bmatrix} a \\ b \end{bmatrix} : \ a \in \mathbb{R}, \ b \geq 0 \}.$$

4. [6 points] Construct an example that does the following. Give a matrix A that is 3×3, a vector $\mathbf{b}$ for which $A\mathbf{x} = \mathbf{b}$ is consistent, and for which Col A is NOT equal to $\mathbb{R}^3$. (Hint: keeping it as simple as possible will be to your advantage.) Justify your answer.

5. [10 points] The following matrix A has been row-reduced to the echelon form B. By examining the 3rd row of B, give a one or two sentence argument that convinces us that the pivot columns of B do not form a basis for the column space of A. Give a basis for the column space of A.

$$A = \begin{bmatrix} 2 & 1 & 1 & 0 \\ 3 & 6 & 1 & 2 \\ 8 & 13 & 3 & 4 \end{bmatrix}$$

$$B = \begin{bmatrix} 1 & 1/2 & 1/2 & 0 \\ 0 & 1 & -1/9 & 4/9 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

State the dimension of the row space, the column space, and the null space.

6. [8 points] Set up (but do not solve) the linear system that allows you to test the linear independence of the given set of polynomials. What theorem/idea are you taking advantage of?

$$1 + 2t^3,\ 2 + t - 3t^2,\ -t + 2t^2 - t^3$$

7. [6 points] **Fill-in-the-blank** If there exists a set $\{\mathbf{v_1}, \ldots, \mathbf{v_p}\}$ that spans the vector space V, then the dimension of V is _____. This is true because we may apply the _____ Theorem to the set $\{\mathbf{v_1}, \ldots, \mathbf{v_p}\}$ so as to produce a _____ for V, which has _____.

8. [8 points] Suppose that a 6×7 matrix A is given. Suppose that the null space of A contains two vectors $\mathbf{u}$ and $\mathbf{v}$ for which $\mathbf{u}$ is NOT a scalar multiple of $\mathbf{v}$. Use the Rank Theorem to argue a bound on the dimension of the column space of A. Why can one only give a bound and not the precise value?