Linear Algebra
Exam 2 - Fall 2017

November 9, 2017

Name:
Honor Code Statement:

Directions: Complete all problems. Justify all answers/solutions. Calculators, cell-phones, texts, and notes are not permitted – the only permitted items to use are pens, pencils, rulers and erasers. Please turn off all electronic devices – in fact, you shouldn't have any with you. Additional blank white paper is available at the front of the room – you are not permitted to use any other paper. Good luck!

1. [10 points] The following is an LU-factorization of A. Use this factorization to solve the matrix equation $A\mathbf{x} = \mathbf{b}$, where $\mathbf{b}$ is given below.

$$L = \begin{bmatrix} 1 & 0 & 0 \\ 1/2 & 1 & 0 \\ 3/2 & -5 & 1 \end{bmatrix}$$

$$U = \begin{bmatrix} 2 & -2 & 4 \\ 0 & -2 & -1 \\ 0 & 0 & -6 \end{bmatrix}$$

$$\mathbf{b} = \begin{bmatrix} 0 \\ -5 \\ 7 \end{bmatrix}$$

2. [8 points] Compute the determinant of the following matrix. Justify your steps.

$$G = \begin{bmatrix} 4 & 8 & 8 & 8 & 5 \\ 0 & 1 & 0 & 0 & 0 \\ 6 & 8 & 8 & 8 & 7 \\ 0 & 8 & 8 & 3 & 0 \\ 0 & 8 & 2 & 0 & 0 \end{bmatrix}$$

[2points] Is G invertible? Why?

3. [10 points] The following set of vectors is not a basis for $\mathbb{R}^3$. Find a basis for $\mathbb{R}^3$ that contains this set. Justify your answer.

$$\mathbf{b_1} = \begin{bmatrix} 2 \\ 2 \\ 2 \end{bmatrix}$$

$$\mathbf{b_2} = \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}$$

4. [5 points] Let $H = \{ \begin{bmatrix} x \\ y \end{bmatrix} : x^2 + y^2 = 0 \textit{ or } x^2 + y^2 = 1 \}$. Is H a subspace of $\mathbb{R}^2$? Why or why not?

5. [5 points] With regard to question 1, compute $\det A$ without computing A.

6. [5 points] Suppose a 5×6 matrix A has four pivot columns.

   (a) What is dim Nul A? Why?

   (b) Is Col $A = \mathbb{R}^4$? Why or why not?

7. [5 points] State the Spanning Set Theorem.

8. [5 points] The following matrix is a block partitioned matrix. Show/prove that the matrix has an inverse AND determine this inverse based upon the block entries.

$$\begin{bmatrix} I_p & 0 \\ A & I_q \end{bmatrix}$$

9. **Fill-in-the-blank** [2 points for each blank] Let $T : \mathbb{R}^n \to \mathbb{R}^m$ be a one-to-one linear transformation. Let us determine the dimension of the range of T.

Let A be the $m \times n$ standard matrix of T. As T is one-to-one, the columns of A are ____. So, we can conclude that the dimension of ____ equals 0. By ____ , dimCol$A$ $= n - 0 = n$, which is the number of columns of ____________________. As the range of T is Col A, the dimension of the range of T is ____________________.