Linear Algebra
Exam 1
Spring 2026

March 16, 2026

Name:
Honor Code Statement:

Directions: Complete all problems. Justify all answers/solutions; notice that some problems have writing limits. Outside sources are not permitted. There is a 90-minute time limit. The exam is proctored with permission of the Dean of the Faculty.

1. [20 points] Consider the following matrix below.

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 1 & 1 & 1 \end{bmatrix}$$

   (a) Consider the homogeneous equation $A\mathbf{x} = \mathbf{0}$. Write the solution set to this equation in parametric vector form and describe the solution set geometrically.

(b) If possible, write the vector $\mathbf{b} = \begin{bmatrix} -2 \\ -1 \\ -2 \end{bmatrix}$ as a linear combination of the columns of A.

(c) Is the mapping $\mathbf{x} \mapsto A\mathbf{x}$ an onto mapping? Why or why not?

(d) Give two vectors in the domain of this mapping that map to the same vector in the codomain. Justify your answer.

2. [5 points] Use the figure below to set up a system of equations that models the general flow pattern of the network given. DO NOT solve this system.

3. [5 points] The following set of vectors form a set that is linearly independent. Indicate why, then give an additional vector to add to the set that would make the set of three linearly dependent. Justify your answer.

$$\mathbf{b_1} = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}, \mathbf{b_2} = \begin{bmatrix} 0 \\ 2 \\ 0 \end{bmatrix}$$

4. [5 points] The following statement is false: "If a set in $\mathbb{R}^3$ is linearly dependent, then the set contains more than 3 vectors." Show that this is false by giving a counter-example.

5. [5 points] Show that T is a linear transformation by finding a matrix that implements the mapping.

$$T(x_1, x_2, x_3, x_4) = (x_1 + x_2, x_2 + x_3, x_3 + x_4, x_4)$$

6. [5 points] Determine whether a square matrix A with two identical rows is invertible. **Two sentence writing limit.**

7. [5 points] Suppose that A is a 2×2 matrix such that $AA = A^2 = I_2$, that is, A is self-inverse. Show that the following block-partitioned matrix is also self-inverse.

$$M = \begin{bmatrix} A & 0 \\ I_2 & -A \end{bmatrix}$$

8. [5 points] **Fill-in-the-blank** Suppose that vectors $\mathbf{v_1}, \mathbf{v_2}, \mathbf{v_3}$ span $\mathbb{R}^3$, and let $T : \mathbb{R}^3 \to \mathbb{R}^3$ be a linear transformation such that $T(\mathbf{v_1}) = T(\mathbf{v_2}) = T(\mathbf{v_3}) = \mathbf{0}$. Then for any $\mathbf{x}$ in $\mathbb{R}^3$, there are constants c_1, c_2, c_3 such that $\mathbf{x} = c_1\mathbf{v_1} + c_2\mathbf{v_2} + c_3\mathbf{v_3}$ because ______________________________. Then we can write

$$T(\mathbf{x}) = c_1T(\mathbf{v_1}) + c_2T(\mathbf{v_2}) + c_3T(\mathbf{v_3})$$

since T preserves ______________________________. Now since $T(\mathbf{v_1}) = T(\mathbf{v_2}) = T(\mathbf{v_3}) = \mathbf{0}$, we have that $T(\mathbf{x})$ equals ________________________. In this way, we might name/describe T as the __________________ transformation.