# Linear Algebra
# Exam 1
# Spring 2017

March 16, 2017

**Name:**
**Honor Code Statement:**

**Additional Statement:** I have not observed another violating the Honor Code.

**Directions:** Complete all problems. Justify all answers/solutions. Calculators, notes or texts are not permitted. Cell phones should not be used at any time (even to check the time) - please put them away! There is a two-hour time limit.

1. [5points] Suppose $T$ is a linear transformation from $\mathbb{R}^3$ to $\mathbb{R}^3$ that projects onto the $x_1, x_2-$plane, that is, $T(x_1, x_2, x_3) = (x_1, x_2, 0)$. Give the standard matrix of the transformation. Also, give a counterexample to the following statement: $T$ is one-to-one.

2. [3 points each] Each of the following statements is false. Amend each statement in as few words or symbols as possible, without insertion or deletion of the word "not" (that is, without negating the conclusion), to make a true statement or explain why it's false, which can be done by pointing to a theorem or giving a counter-example.

   (a) If every column of an augmented matrix contains a pivot, then the corresponding system is consistent.

   (b) If the coefficient matrix $A$ has a pivot position in every row, then the equation $A\mathbf{x} = \mathbf{b}$ is inconsistent.

   (c) If $\mathbf{x}$ is a nontrivial solution of $A\mathbf{x} = \mathbf{0}$, then every entry in $\mathbf{x}$ is nonzero.

   (d) If $T : \mathbb{R}^n \to \mathbb{R}^m$ is a linear transformation and if $\mathbf{c}$ is in $\mathbb{R}^m$, then a uniqueness question is "Is $\mathbf{c}$ in the range of T?"

   (e) Suppose that $\mathbf{v_1}, \mathbf{v_2}, \mathbf{v_3}$ are in $\mathbb{R}^5$, $\mathbf{v_2}$ is not a multiple of $\mathbf{v_1}$ and $\mathbf{v_3}$ is not a linear combination of $\mathbf{v_1}$ and $\mathbf{v_2}$. Then $\{\mathbf{v_1}, \mathbf{v_2}, \mathbf{v_3}\}$ is linearly independent.

3. [5 points] Limestone, $CaCO_3$, neutralizes the acid, $H_3O$, in acid rain by the following unbalanced equation:

$$H_3O + CaCO_3 \to H_2O + Ca + CO_2.$$

Write a vector equation that demonstrates how one would balance this equation. (You need **not** solve this equation.)

4. [10 points] Suppose that $T : \mathbb{R}^4 \to \mathbb{R}^3$ is an onto linear transformation. Describe the *four* different possible echelon forms of the standard matrix for $T$, where $\blacksquare$ corresponds to a pivot entry, and $*$ corresponds to any real number.

**True or False**: T can be one-to-one.

5. [5 points] Show that the following matrix does not have an inverse.

$$M = \begin{bmatrix} 2 & 3 & -1 \\ -8 & -7 & 6 \\ 6 & -1 & -7 \end{bmatrix}$$

6. [5 points] Define what it means for a transformation $T$ from $\mathbb{R}^n$ to $\mathbb{R}^m$ to be **onto**.

7. [10 points total] **Construct an example:** In each of the following prompts give an example of the desired object(s) **and** *demonstrate/prove* that these objects have the desired property.

   (a) A $2 \times 2$ matrix $A$ and a $2 \times 2$ matrix $B$ which do not commute.

   (b) An $3 \times 3$ elementary matrix whose inverse has the entry 1 in row 1, column 2.

   (c) An invertible $4 \times 4$ matrix for which precisely one calculation (e.g. elementary row operation, matrix multiplication, etc.) is needed to demonstrate its invertibility.

8. [5 points] Suppose $\{\mathbf{v_1}, \mathbf{v_2}\}$ is a linearly independent set in $\mathbb{R}^n$. Show that $\{\mathbf{v_1}, \mathbf{v_1} + \mathbf{v_2}\}$ is also a linearly independent set.