Linear Algebra
Exam 1
Spring 2014

March 13, 2014

Name:
Honor Code Statement:

Directions: Complete all problems. Fill-in-the-blank problems are worth 2 points each and all others are worth 5 points each. Justify all answers/solutions. Calculators, notes or texts are not permitted. There is a two-hour time limit.

1. Use Gaussian Elimination to solve the following linear system. Identify pivot columns. Identify free variables. Express your answer in parametric vector form.

$$x_1 - 3x_2 - 5x_3 = 0$$
$$x_1 - 2x_2 - 4x_3 = 3$$

2. Compute the following product: $A\mathbf{x}$, where $A = \begin{bmatrix} 1 & 1 & 0 & 5 \\ -1 & 1 & -1 & 11 \\ 2 & 0 & 2 & 6 \end{bmatrix}$ and

$$\mathbf{x} = \begin{bmatrix} 1 \\ 2 \\ -1 \\ 0 \end{bmatrix}.$$

3. In regards to the matrix in the previous problem, what does the size of A tell you about its invertibility?

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4. Find the inverse of the following elementary matrix.

$$M = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix}$$

5. Give 5 statements that are equivalent to the following: an $n \times n$ matrix A is *not* an invertible matrix.

6. Define the *subset of* $\mathbb{R}^n$ *spanned by* $\{\mathbf{v_1}, \ldots, \mathbf{v_p}\}$.

7. Geometrically, for a non-zero vector $\mathbf{v}$, Span$\{\mathbf{v}\}$ is __________________________.
Geometrically, for the zero vector $\mathbf{0}$, Span$\{\mathbf{0}\}$ is __________________________.

8. The following transformation, which maps vectors from $\mathbb{R}^2$ to vectors in $\mathbb{R}^2$, is not onto: $\mathbf{x} \mapsto A\mathbf{x}$, where $A = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$.

   Give a vector in the codomain that has no pre-image to demonstrate that this is so.

9. The following transformation, which maps vectors from $\mathbb{R}^2$ to vectors in $\mathbb{R}^2$, is not one-to-one: $\mathbf{x} \mapsto A\mathbf{x}$, where $A = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$.

   Give a vector in the codomain and two of its pre-images to demonstrate that this is so.

10. In regards to the previous problem: (fill-in-the-blank) we know that this transformation is not one-to-one by examining the columns of A – this set of columns ____.

11. Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be a linear transformation. The illustration below shows $T(\mathbf{e_1})$ and $T(\mathbf{e_2})$. Accurately sketch $T(2, 3)$.

12. Consider the following network flow problem as illustrated below. *Assuming* that there is a solution (i.e. you needn't do any Gaussian elimination) to the flow problem, is there another? Why?

13. The following transformation from $\mathbb{R}^3$ to $\mathbb{R}^3$ is not a linear transformation: $T(\mathbf{x}) = 3\mathbf{x} + \mathbf{v}$, where $\mathbf{v} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$. Give a brief argument to support the claim that T is not a linear transformation.

14. The solution set of the matrix equation $A\mathbf{x} = \mathbf{0}$ is illustrated below, where A is a 2×2 matrix. Suppose that for some non-zero $\mathbf{b}$ the equation $A\mathbf{x} = \mathbf{b}$ has the solution $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$. Draw the entire solution set for $A\mathbf{x} = \mathbf{b}$.

15. In the above problem, both the domain and codomain of the transformation (of multiplication by A) are $\mathbb{R}^2$. But, does the illustration above represent the domain or the codomain? It represents the __________________________.