Linear Algebra
Exam 1
Spring 2013

March 14, 2013

Name:
Honor Code Statement:

Additional Statement: I have not observed another violating the Honor Code.

Directions: Complete all problems. Justify all answers/solutions. Calculators, notes or texts are not permitted. There is a two-hour time limit.

1. [3 points each] Each of the following statements is false. Amend each statement in as few words or symbols as possible, without insertion or deletion of the word "not" (that is, without negating the conclusion), to make a true statement or explain why it's false.

   (a) A homogeneous system of equations can be inconsistent.

   (b) When $\mathbf{u}$ and $\mathbf{v}$ are nonzero vectors, $Span\{\mathbf{u}, \mathbf{v}\}$ contains only the line through $\mathbf{u}$ and the origin, and the line through $\mathbf{v}$ and the origin.

   (c) If a set in $\mathbb{R}^n$ is linearly dependent, then the set contains more than n vectors.

   (d) A mapping $T : \mathbb{R}^n \to \mathbb{R}^m$ is onto $\mathbb{R}^m$ if every vector $\mathbf{x}$ in $\mathbb{R}^n$ maps onto some vector in $\mathbb{R}^m$.

2. [8 points] Find the inverse of the following matrix.

$$M = \begin{bmatrix} -2 & -1 & -2 \\ -1 & 1 & -1 \\ 1 & -2 & 3 \end{bmatrix}$$

3. [6 points] Using the matrix M from the previous problem, re-write the matrix equation $M\mathbf{x} = \mathbf{b}$, where $\mathbf{b} = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$, as a system of linear equations *and* as a vector equation.

4. [3 points] State what it means for a set of vectors to *span* $\mathbb{R}^3$.

5. [10 points]The linear transformation $T_1 : \mathbb{R}^2 \to \mathbb{R}^2$ rotates points through $\frac{\pi}{4}$ radians in a counterclockwise fashion. What is the matrix transformation that achieves this?

The linear transformation $T_2 : \mathbb{R}^2 \to \mathbb{R}^2$ maps $\mathbf{e_1}$ to itself and $\mathbf{e_2}$ to the vector $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$. What is the matrix transformation that achieves this?

Using matrix algebra, show that the $T_1(T_2(\mathbf{x})) \neq T_2(T_1(\mathbf{x}))$ for some $\mathbf{x}$ in $\mathbb{R}^2$.

6. [4 points] Find the parametric equation of the line through $\mathbf{a} = \begin{bmatrix} -2 \\ 0 \end{bmatrix}$ and parallel to $\mathbf{b} = \begin{bmatrix} -5 \\ 3 \end{bmatrix}$.

7. [4 points] To reduce a matrix to reduced echelon form, there are two phases, the forward phase and the backward phase. If you had to do one of these phases (by hand) for a 25×25 matrix and your best friend had to do the other, which would you choose and why?

8. [8 points] Let $T : \mathbb{R}^n \to \mathbb{R}^m$ have the form $T(\mathbf{x}) = A\mathbf{x} + \mathbf{b}$, with A an $m \times n$ matrix and $\mathbf{b}$ in $\mathbb{R}^m$. (Such a transformation is called an *affine transformation.*) Suppose that $\mathbf{b} = \mathbf{e_1}$, is T linear? Justify your answer.

9. [5 points] Given the flow pattern of the network below and without solving the associated linear system, if the flow pattern has a solution, is there a second solution? Justify your answer.