Linear Algebra
Exam 1
Fall 2023

October 19, 2023

Name:
Honor Code Statement:

Directions: Complete all problems. Justify all answers/solutions; notice that some problems have writing limits. Outside sources are not permitted. There is a 90-minute time limit.

1. [5 points] Find the entry in row 2, column 1 of the product AB, where A and B are given below. Then find the entry in row 2, column 1 of the product BA.

$$A = \begin{bmatrix} 6 & 1 & 0 \\ -1 & 1 & 1 \\ 0 & 0 & 5 \end{bmatrix}$$

$$B = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & -10 \\ 2 & 0 & 2 \end{bmatrix}$$

This calculation demonstrates that matrix multiplication is ____________________.
(Fill-in-the-blank.)

2. [10 points] Determine if the vector $\mathbf{b}$ is a linear combination of the vectors $\mathbf{a_1}, \mathbf{a_2}, \mathbf{a_3}$.

$$\mathbf{b} = \begin{bmatrix} -2 \\ -1 \\ -2 \end{bmatrix}, \mathbf{a_1} = \begin{bmatrix} 0 \\ -2 \\ 0 \end{bmatrix}, \mathbf{a_2} = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}, \mathbf{a_3} = \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}$$

Is the set $\{\mathbf{a_1}, \mathbf{a_2}, \mathbf{a_3}\}$ a spanning set for $\mathbb{R}^3$? Why or why not?

3. [5 points] Using this same set of vectors $\{\mathbf{a_1}, \mathbf{a_2}, \mathbf{a_3}\}$ as in the previous problem, can you write $\mathbf{a_3}$ as a linear combination of the others? If so, give one such linear dependence relation. (You may rely on previous calculations, if you wish.)

4. [5 points] Give a 3×3 matrix B such that the map $\mathbf{x} \mapsto B\mathbf{x}$ is not a one-to-one mapping. Demonstrate that the mapping is many-to-one by giving two distinct vectors $\mathbf{u}$ and $\mathbf{v}$ for which $B\mathbf{u} = B\mathbf{v}$.

5. [5 points] Set up but do not solve the linear system that arises from the following network under the assumption of preservation of flow (i.e. "flow in equals flow out").

6. [5 points] Consider the transformation T defined by $T(x_1, x_2) = (x_1^2, x_2)$. Show that the transformation fails to have either property of being a *linear* transformation.

7. [5 points] Let A be a 100×100 invertible matrix. Give the algorithm for finding its inverse. Approximately how many flops are required to find this inverse. (Three sentence writing limit.)

8. [5 points] Give the standard matrix of the transformation T that maps from $\mathbb{R}^2$ to $\mathbb{R}^2$ that rotates points around the origin by 180-degrees clockwise. Is the matrix you found invertible?

9. [5 points] Remember that our goal is to solve $A\mathbf{x} = \mathbf{b}$. Suppose that A has an LU-factorization. Describe how we use this LU-factorization to solve $A\mathbf{x} = \mathbf{b}$. (Four sentence writing limit.)