Linear Algebra
Exam 1
Fall 2021

October 14, 2021

Name:
Honor Code Statement:

Directions: Complete all problems. Justify all answers/solutions. Calculators, notes or texts are not permitted. There is a two-hour time limit.

1. [5 points]The linear transformation $T : \mathbb{R}^2 \to \mathbb{R}^2$ that projects points onto the x_1 axis can be realized as a matrix transformation. What is the matrix transformation that achieves this?

2. [5 points] **Define** what it means for a set of vectors to be **linearly independent**. Give an example of a linearly independent set of vectors in $\mathbb{R}^3$ consisting of three vectors.

3. [3 points each] Each of the following statements is false. Amend each statement in as few words or symbols as possible to make a true statement, without simply negating the conclusion.

(a) The follow matrix is an example of an elementary matrix, $E = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 4 \end{bmatrix}$.

(b) If A is an $n \times n$ non-singular matrix, then the columns of A form a linearly dependent set.

(c) If $AB = C$ and C has 3 rows and 2 columns, then A has 3 rows and B has 2 rows.

4. [8 points] Find the inverse of the following matrix.

$$M = \begin{bmatrix} 1 & 1 & 0 \\ -1 & 1 & -1 \\ 2 & 0 & 2 \end{bmatrix}$$

5. [5 points] Use the inverse from the following problem to solve the matrix equation $M\mathbf{x} = \mathbf{b}$, where b is the following column vector,

$$\mathbf{b} = \begin{bmatrix} 5 \\ 5 \\ 1 \end{bmatrix}.$$

6. [5 points] Find the following product MA, where M is given in a previous exercise and A is given below.

$$A = \begin{bmatrix} 2 & 1 & 0 \\ 3 & 1 & 1 \\ 0 & 0 & 5 \end{bmatrix}$$

7. [5 points] Given the flow pattern of the network below, write a linear system that would allow you to determine the general flow pattern. (You don't need to solve the linear system.)

8. [10 points] The solution set of a homogeneous linear system in three variables is given. Write the solution set in parametric vector form. Give a geometric description of the solution set.

$$x_1 + -5x_3 = 0$$

$$x_2 - 8x_3 = 0$$

$$x_3 \; is \; free$$

Suppose that a non-homogeneous linear system is given with the same coefficient matrix as this one (though note, I haven't given you the coefficient matrix). What can you say about the solution set of this system in relation to the one above?

9. [4 points] Let A be an $n \times n$ matrix. The following statements are not all equivalent. There is one statement which can be removed and we'd be left with an equivalent set of statements. Which one is it? Why?

   (a) A has fewer than n pivots.

   (b) A is singular.

   (c) The columns of A do not span $\mathbb{R}^n$.

   (d) The equation $A\mathbf{x} = \mathbf{0}$ has more than one solution.

   (e) The columns of A do not form a linearly dependent set.

10. [8 points] **Fill in the four (4) blanks** Suppose that $CA = I_n$. Show that the equation $A\mathbf{x} = \mathbf{0}$ has only the trivial solution.

PROOF: If $\mathbf{x}$ satisfies $A\mathbf{x} = \mathbf{0}$, then $CA\mathbf{x} = C\mathbf{0} = \mathbf{0}$ and so $I_n\mathbf{x} = \mathbf{0}$ and $\mathbf{x} = \mathbf{0}$. This shows that the equation _____________ has no _____________. So every variable is a_____________ and every column of A is a _____________ column.

11. [6 points] **Fill in the three (3) blanks** If a set contains more vectors than there are entries in each vector, then the set is linearly dependent. That is, any set $\{\mathbf{v_1}, \ldots, \mathbf{v_p}\}$ in $\mathbb{R}^n$ is linearly dependent if $p > n$.

PROOF: Let $A = [\mathbf{v_1} \cdots \mathbf{v_p}]$. Then A is $n \times p$, and the equation $A\mathbf{x} = \mathbf{0}$ corresponds to a system of n ____________________ in p unknowns. If $p > n$, there are more variables than equations, so there must be a ____________________. Hence $A\mathbf{x} = \mathbf{0}$ has a ___________________________ and the columns of A are linearly dependent.