Linear Algebra Exam 1 Fall 2014

October 9, 2014

Name:

Honor Code Statement:

Directions: Complete all problems. Fill-in-the-blank problems are worth 2 points each and all others are worth 5 points each unless otherwise noted. Justify all answers/solutions. Calculators, notes or texts are not permitted. There is a two-hour time limit.

1. [10 points] Use Gaussian Elimination to solve the following linear system. Identify pivot columns. Identify free variables. Express your answer in parametric vector form.

$$2x_1 + 0x_2 - 6x_3 = -8$$
$$0x_1 + x_2 + 2x_3 = 3$$
$$3x_1 + 6x_2 - 2x_3 = -4$$

2.	[5 points] Rewrite the linear system equation and a matrix equation.	from	the	previous	question	as	both	a	vector

3. [5 points] A flop is one arithmetic operation. A flip is turning over one pancake. Suppose that the time it takes for a flop is the same as a flip. Which takes less time, flipping pancakes for each member of the student body (where each student gets 3 pancakes) or the forward phase of Gaussian Elimination on a 50×50 -matrix.

4. [5 points] Give an example of a consistent linear system with *more* equations than unknowns.

5. [5 points] Give a 3×3 matrix, not in echelon form, whose columns do *not* span \mathbb{R}^3 . Justify your answer.

6.	[3 points each] Each	of the following	statements is false	Amend each	statement in
	as minimal a way as	possible so as to	create a true state	ment.	

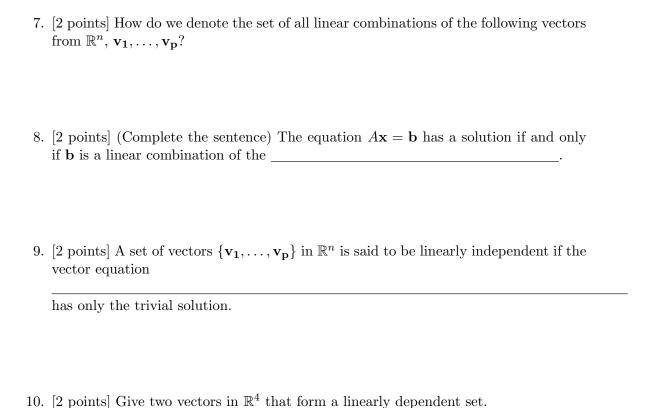
(a) If every column of an augmented matrix contains a pivot, then the corresponding system is consistent.

(b) The vector \mathbf{v} results when a vector $\mathbf{u} - \mathbf{v}$ is added to the vector \mathbf{v} .

(c) If A is an $m \times n$ matrix whose columns do not span \mathbb{R}^m , then the equation $A\mathbf{x} = \mathbf{b}$ is consistent for every \mathbf{b} in \mathbb{R}^m .

(d) If \mathbf{x} is a nontrivial solution of $A\mathbf{x} = \mathbf{0}$, then every entry in \mathbf{x} is nonzero.

(e) A product of invertible $n \times n$ matrices is invertible, and the inverse of the product is the product of their inverses in the same order.



11. [5 points] Assume that T is a linear transformation. Find the standard matrix of T. T maps from \mathbb{R}^2 to \mathbb{R}^2 by rotating points counter-clockwise about the origin through $3\pi/2$ radians.

12.	[2 points for each blank] Let us prove the following theorem.
	Theorem Any set $\{\mathbf{v_1}, \dots, \mathbf{v_p}\}$ in \mathbb{R}^n is said to be linearly dependent if $p > n$.
	PROOF: Let $A = [\mathbf{v_1}, \dots, \mathbf{v_p}]$. Then A is $n \times p$, and the equation $A\mathbf{x} = 0$ corresponds to a system of If $p >$
	n, there are more than equations, so
	there must be a Hence $A\mathbf{x} = 0$ has
	a, and the columns of A are linearly
	dependent. \square

13. [4 points] The following statement is false: If $\mathbf{v_1}, \mathbf{v_2}, \mathbf{v_3}$ are in \mathbb{R}^3 and $\mathbf{v_3}$ is not a linear combination of $\mathbf{v_1}, \mathbf{v_2}$, then $\{\mathbf{v_1}, \mathbf{v_2}, \mathbf{v_3}\}$ is linearly independent. Provide a counterexample to show that it is false.

 $14.\ [5\ points]$ Prove that a linear transformation maps the zero vector to the zero vector.