Problem Sets

Graph Theory - MATH 247

May 4, 2021

Problem sets due at 5pm of the day indicated via the Canvas site. *Please note that the first printing of the text has some typographical errors. I'll try to alert these to you when I know about them.*

1. Due Wednesday, March 3
   Read: Syllabus, Thoughts on Homework
   Read: textbook's Preface, Prologue, Chapter 1
   Turn in (via Canvas site): Problems 2, 7, 9, 12, 19
   Note: In drawing the graphs from these problems, please draw such that structural characteristics are easily visible.

2. Due Wednesday, March 10
   Read: Chapter 2; Handout on Mathematical Induction (see the Google folder); and first page of "Vertex-coloring edge-weightings: Towards the 1-2-3-conjecture" by Kalkowski, Karoński and Pfender
   Turn in: Problems 2, 7, 8, 13, 19

   Also do: Problem 6, Prove Theorem 2.3 using induction on the number of distinct elements in the set

3. Due Wednesday, March 17
   Read: Chapter 3 and "Merlin's Magic Square" by Don Pelletier (which is available in the Google folder)
   Turn in: Problems 5, 13, 17, 21, 23

   Install software on a laptop so that you can use LaTeX. See the instructions on the course webpage. If this is not possible, please let me know. You will be asked to use LaTeX to typeset at least one problem from each future problem set, including this one.

   Also do: Problem 22; 25; Prove that every graph with no odd cycles is bipartite using induction on the number of vertices.

4. Due Wednesday, March 24
   Read: Chapter 4
   Turn in: Problems 5, 10, 11(a),(c), 14. (At least one of these problems must be typeset using LaTeX.)

   Also do: 4, 11(b), 11(d), 15, find the Prüfer code of a given labelled tree, given a Prüfer code draw the labelled tree

5. Due Friday, April 2
   Read: Chapter 5
   Turn in: 2, 8, 13, 17

   Also do: Apply Dijkstra's algorithm to a weighted graph; know the definition of De Bruijn graphs and what the first few look like; State and prove a characterization of Eulerian digraphs (recall the definition of strongly-connected);

   This article about de Bruijn graphs and genome assembly is interesting:
   https://www.ncbi.nlm.nih.gov/pmc/articles/PMC5531759/

6. Due Friday, April 16
   Read: Chapter 6
   Turn in: 6, 9, 12, Prove Theorem 6.3
   Also do: 10; Prove that Theorem 6.2 is a corollary of Theorem 6.3; Characterize when the graph $K_{n_1,n_2,\ldots,n_p}$ is hamiltonian (and justify the characterization, of course!); Prove that the Petersen graph is not hamiltonian; Prove that Q_4 is hamiltonian; Give an example of a graph on 6 vertices that shows that the greedy approach to TSP can be arbitrarily bad

7. Due Friday, April 16
   Read: Chapter 7
   Turn in: 1, 4, 9, 15 *(For problem 15: the first printing of the text doesn't include the edges u_2u_3 and v_2v_3, so please insert these.)*, and 16

8. Due Monday, April 26
   Read: Chapter 8
   Turn in: 1, 7 (for part (a) also consider P_6 and P_7), 11, 13
   Also do: 6

9. Due Wednesday, May 5
   Read: Chapter 10
   Turn in: 4, 7, 11, 15
   Also do: 5, 6, 8, 12, 17; Give (and obviously prove) upper bounds for the crossing number of $K_{m,n}$ - one upper bound where "kilns" and "sheds" are each on their own line and the lines are parallel, and a better upper bound where "kilns" and

"sheds" are each on their own line and the lines are perpendicular and intersecting (as Zarankiewicz did in 1954)

10. Due Friday, May 14
    Read: Chapter 11
    Turn in: 5,8,9,10
    Also do: 7, 12; Construct a graph G that is neither a complete graph nor an odd cycle but has a vertex ordering relative to which greedy coloring uses $\delta(G)+1$ colors; Prove or disprove: For every graph G, $\chi(G) \leq n(G) - \alpha(G) + 1$.