Problem Sets

Graph Theory - MATH 247

November 24, 2019

Please note that the first printing of the text has some typographical errors. I'll try to alert these to you when I know about them.

1. Due Friday 9/13
   Read: Preface, Prologue, Chapter 1
   Turn in: Problems 2, 7, 9, 12, 19
   Note: In drawing the graphs from these problems, please draw such that structural characteristics are easily visible.

2. Due Friday 9/20
   Read: Chapter 2 and "Vertex-coloring edge-weightings: Towards the 1-2-3-conjecture" by Kalkowski, Karoński and Pfender
   Turn in: Problems 2, 7, 8, 13, 19

   Also do: Problem 6, Prove Theorem 2.3 using induction on the number of vertices

3. Due Friday 9/27
   Read: Chapter 3 and "Merlin's Magic Square" by Don Pelletier (which you can find via MathSciNet)
   Turn in: Problems 5, 13, 17, 21, 23

   Apply the algorithm of Kalkowski, Karoński and Pfender to the graph made by an 8-cycle together with a chord joining two vertices at distance 2 along the cycle. Show the steps of the algorithm, that is, give the labellings of the graph as well as the final labelling.

   Install software on a laptop so that you can use LaTeX. See the instructions on the course webpage. If this is not possible, please let me know. You will be asked to use LaTeX to typeset at least one problem from each future problem set.

   Also do: Problem 22, 25, Prove that every graph with no odd cycles is bipartite using induction on the number of vertices.

4. Due Friday 10/4
   Read: Chapter 4
   Turn in: Problems 5, 10, 11(a),(c), 14, 19. (At least one of these problems must be typeset using LaTeX.)

   Also do: 4, 11(c), 11(d), 15, find the Prüfer code of a given labelled tree, given a Prüfer code draw the labelled tree

5. Due Friday 10/18
   Read: Chapter 5
   Turn in: 2, 8, 13, 17

   Also do: Apply Dijkstra's algorithm to a weighted graph; know the definition of De Bruijn graphs and what the first few look like; State and prove a characterization of Eulerian digraphs (recall the definition of strongly-connected);

6. Due Friday 10/25
   Read: Chapter 6
   Turn in: 6, 9, 13, Prove Theorem 6.3
   Also do: 10; Prove that Theorem 6.2 is a corollary of Theorem 6.3; Characterize when the graph $K_{n_1,n_2,\ldots,n_p}$ is hamiltonian (and justify the characterization, of course!); Prove that the Petersen graph is not hamiltonian; Prove that Q_4 is hamiltonian without referring to a drawing and without listing out the order one visits all 16 vertices; Give an example of a graph on 6 vertices that shows that the greedy approach to TSP can be arbitrarily bad

7. Due Friday 11/1
   Read: Chapter 7
   Turn in: 1, 4, 9, 15 [1], 16

8. Due Friday 11/8
   Read: Chapter 8
   Turn in: 1, 7 (for part (a) also consider P_6 and P_7), 11, 13
   Also do: 6

   Exam 2, Part 1 will occur on November 11th during class, with a "choose 3 of 4" format and covering chapters 5 and 6. Exam 2, Part 2 will occur on November 13th during class, with a "choose 3 of 4" format and covering chapters 7 and 8.

   On Friday, November 15th a class member, Tommaso Monaco, will lecture about a graph theory problem he worked on this past summer. Attendance at this lecture will facilitate you being able to complete the next assignment.

[1] The first printing of the text doesn't include the edges u_2u_3 and v_2v_3, so please insert these.

9. Due Wednesday 11/20
   Read: Section 1 and 3 of A. Hertz and C. Picouleau, *On graceful difference labelings of disjoint unions of circuits* available at `https://arxiv.org/abs/1908.11300`.
   Turn in: To each vertex v in such a graph described in the article (you can pick a particular graph, if you like) associate a variable x_v. Then give a polynomial that associates solutions of this problem with the non-zeros of the polynomial and verify this. To do this, you will need to have attended Tommaso's lecture on Friday the 15th. Any additional information about this polynomial (such as the coefficient on one of its monomials) is welcome. [30 points for one problem]

10. Due Tuesday 11/26 at noon
    Read: Chapter 10
    Turn in: 4, 7, 11, 15
    Also do: 5, 6, 8, 12, 17; Give (and obviously prove) upper bounds for the crossing number of $K_{m,n}$ - one upper bound where "kilns" and "sheds" are each on their own line and the lines are parallel, and a better upper bound where "kilns" and "sheds" are each on their own line and the lines are perpendicular and intersecting (as Zarankiewicz did in 1954)

11. Due Friday 12/6
    Read: Chapter 11
    Turn in: 5,8,9,10
    Also do: 7, 12

    Final exam will cover Chapters 10 and 11 and the article of Hertz and Picouleau is on Saturday, December 14th, 2-5pm.