Problem Sets

Graph Theory - MATH 247

December 3, 2017

1. Due Friday, 9/15
   Turn in: Section 1.1 numbers 5, 7, 10, 12
   Also do: Section 1.1 numbers 3, 6, 9, 13,18, 29, 33, 36

2. Due Friday, 9/22
   Turn in: Section 1.2 numbers 2, 6, 16, 33
   Also do: Section 1.2 numbers 1, 8, 9, 20, 24

3. Due Monday 10/2
   Turn in Section 1.3 numbers 3, 10, 12, 21, 57
   Also do: Section 1.3 numbers 1, 8(a), 8(b), 14, 42, 58

4. Not collected, but consider ahead of exam on Thursday
   Section 1.4 numbers 3, 5, 8, 19, 36
   Also do: Section 1.4 number 2, 6, 26, 32

5. Due Monday 10/16
   Give two questions of your own making that arise as the result of Friday's discussion about distance, diameter, radius and eccentricity.

6. Due Friday 10/27
   Turn in your favorite from this list (and the others become "also do") Section 2.1 numbers 4, 13, 21, 47 and Section 2.2 numbers 10, 23, 29
   Also do: Section 2.1 numbers 1, 2, 12, 24, 28 and Section 2.2 numbers 1, 2, 5, 7, 20 and (simply) read 35

7. Due Friday 11/3
   Turn in Section 2.3, 2, 4, 13
   Also do: Section 2.3 - 3, 10

8. Due Monday 11/6
   Turn in Section 3.1 - 5, 8, 18
   Also do Section 3.1 - 2, 16, 20, 27

9. Due Friday 11/17

   1) Let e be a cut-edge in a connected graph G having a perfect matching M. Prove that $e \in M$ if and only if $G - e$ consists of two components with odd order. Conclude that if every vertex of G has odd degree, then every perfect matching in G contains all cut-edges of G.

   2) Let G be a simple graph with $\alpha'(G) = m$. Prove that G has at most $m(m-1)$ edges that belong to no maximum matching. Construct examples to show that this bound is best possible for every m. (Fred Galvin)

   Also do: Section 3.3 1, 3, 6, 15

10. Due Friday 12/1

    1) Construct a graph with nine vertices having an optimal coloring that cannot arise via greedy coloring.

    2) The integer simplex (or triangular grid) with dimension d and side-length m is the graph T_m^d whose vertices are the nonnegative integer $(d+1)$-tuples summing to m, with two vertices adjacent when they differ by 1 in two places and are equal in all other places. Determine $\chi(T_m^d)$.

    3) Prove or disprove: For every graph G, $\chi(G) \leq \omega(G) + \frac{n(G)}{\alpha(G)}$.

    Also do: Section 5.1 - 7, 9, 14, 36, 49, Section 5.2 - 1, 2, 7, 10, 18

11. Due Friday 12/8

    1) The rhombicosidodecahedron is a polyhedron in which every vertex is incident to one triangular face, one pentagonal face, and two (opposite) quadrilateral faces. Determine the number of faces in the rhombicosidodecahedron.

    2) Let G be the graph obtained from the 3-dimensional cube Q_3 by adding an edge with endpoints $(0,0,0)$ and $(1,1,1)$. Find a planar embedding of G or show that it is nonplanar by using Kuratowski's Theorem.

    3) Let G be a triangulation with at least four vertices. Letting n_i be the number of vertices of degree i, prove that $3n_3 + 2n_4 + n_5 \geq 12$.

    Also do: Section 6.1 - 3, 8, 11; Section 6.2 - 2(a), 2(b), 5; Also do: Section 6.1 2, 27, 30 , Section 6.3 - 4, 5, 6