Graph Theory - MATH 247

Exam 3

Name:
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Signature

Directions: Please complete all but 1 problem. There is a time limit of 3 hours.

1. Let G be a simple n-vertex graph with vertex degrees $d_1 \leq d_2 \leq \ldots \leq d_n$. Prove that if $d_j \geq j+1$ whenever $j \leq n-1-d_{n-1}$, then G is 2-connected.

2. Given a graph G with vertex set $v_1, v_2, \ldots, v_n$, let G' be the graph generated from G by Mycielski's construction. Let H be a subgraph of G. Let G'' be the graph obtained from G' by adding the edges $\{u_i u_j : v_i v_j \in E(H)\}$. Prove that $\chi(G'') = \chi(G) + 1$ and that $\omega(G'') = max\{\omega(G), \omega(H) + 1\}$.

3. Use the Four Color Theorem to prove that every planar graph decomposes into two bipartite graphs. What further can you say if the condition color-critical is also imposed?

4. A graph is **outerplanar** if it has an embedding with every vertex on the boundary of the unbounded face. An **outerplane graph** is such an embedding of an outerplanar graph. For $n \geq 2$, determine the maximum number of edges in a simple outerplane graph with n vertices by *using* Euler's Formula. (Hint: Consider the length of the unbounded, i.e. the outside, face.)

5. Determine if the graph obtained from K_6 by deleting a perfect matching is planar.

6. Let G be a connected, cubic graph of order $n > 4$ having girth 3. Determine $\chi(G)$.

7. Given that $e(T_{n,r}) = (1 - \frac{1}{r})\frac{n^2}{2} - \frac{b(r-b)}{2r}$ where $a = \lfloor \frac{n}{r} \rfloor, b = n - ra$ and $T_{n,r}$ denotes the Turán graph, can the chromatic number of a graph on 20 vertices and 181 edges be 10? Why, or why not?