Graph Theory - MATH 247

Exam 3

December 12, 2024

Name:
Honor Code Pledge:

Signature:

Directions: Complete 3 of 4 problems; the remainder of the final exam is your group project to be submitted on the Canvas page. There is a time limit of 3 hours.

Best of luck!

Thanks for your work and attention throughout the semester. Peace, J.

1. (a) Show that the Petersen graph is not planar.

    (b) When I asked ChatGPT if there is a 4-regular planar graph of order 9 it gave an incomplete (somewhat inaccurate) description of an explicit example as given below. Leverage the idea presented to draw a 4-regular planar graph of order 9 by giving a planar drawing of such a graph.

    Example Construction: An explicit example of such a graph is the 4-regular planar graph obtained from a subdivision of the 3-cube (cube graph). The 3-cube has 8 vertices and 12 edges; adding one vertex and connecting it appropriately can create a 4-regular planar graph of order 9.

2. Show that $K_{1,5}$ is graceful. Use this graceful labeling to show that K_{10} is $K_{1,5}$-decomposable. (Notice that Theorem 8.6, that we discussed, would apply to a cyclic decomposition of K_{11} – I'm not asking for a cyclic decomposition of K_{11}.). Illustrate the decomposition as simply as possible.

3. Solve the given Instant Insanity Puzzle below. A figure that lists all 17 possible 2-regular multi-graphs is attached at the end of the exam. Upon completing the exam, you will be asked to stack the cubes that are sitting on my desk – your grade will be determined based upon this physical stacking of cubes.
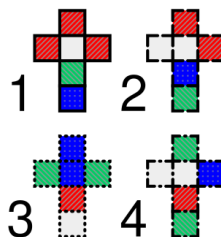

Figure 1: Instant Insanity Puzzle

4. The traditional design of a soccer ball is in fact a (spherical projection of a) truncated icosahedron. This consists of 12 regular pentagons and 20 regular hexagons. No two pentagons are adjacent (so the edges of each pentagon are shared only by hexagons). How many vertices, edges, and faces does a truncated icosahedron have? Explain how you arrived at your answers.