Graph Theory - MATH 247

Exam 3

December 14, 2019

Name:
Honor Code Pledge:

Signature:

Directions: Complete 6 of 7 problems. There is a time limit of 3 hours.
Best of luck!

Thanks for your work and attention throughout the semester. Peace, J.

1. Show that the Petersen graph does not contain a subdivision of K_5. Then show that the Petersen graph is nonplanar.

2. Recall that $\overrightarrow{C_3}$ and $\overrightarrow{C_2} + \overrightarrow{C_3}$ do not have graceful difference labelings. Give a constructive (i.e. do NOT use a polynomial method) proof that shows that $2\overrightarrow{C_2}$ and $2\overrightarrow{C_2} + \overrightarrow{C_3}$ have graceful difference labelings.

3. State and prove the best-known upper bound for the rectilinear crossing number (i.e. Turán's Brick-Factory problem with straight line tracks) as given by Zarankiewicz for the graph $K_{6,15}$. Be sure to describe the layout of the vertices. (Note: Drawing all 90 edges of this graph to determine this number is NOT what I'd hope you would do.)

4. Show that every planar graph of order at least 4 has at least 4 vertices of degree less than or equal to 5.

5. For the graph G below, find the chromatic number $\chi(G)$. Then provide an ordering of the vertices for which the greedy coloring algorithm uses more than $\chi(G)$ colors. Show the coloring that the greedy algorithm returns for your chosen ordering.

6. A planar graph is **outerplanar** if it has an embedding with every vertex on the boundary of the unbounded face. Some examples are given below. Use the Four Color Theorem to prove that every such graph is 3-colorable (i.e. has chromatic number at most 3).

7. Construct a 4-regular n-vertex graph with chromatic number 4. The larger the value of n (and where G is connected), the more points you will be awarded.

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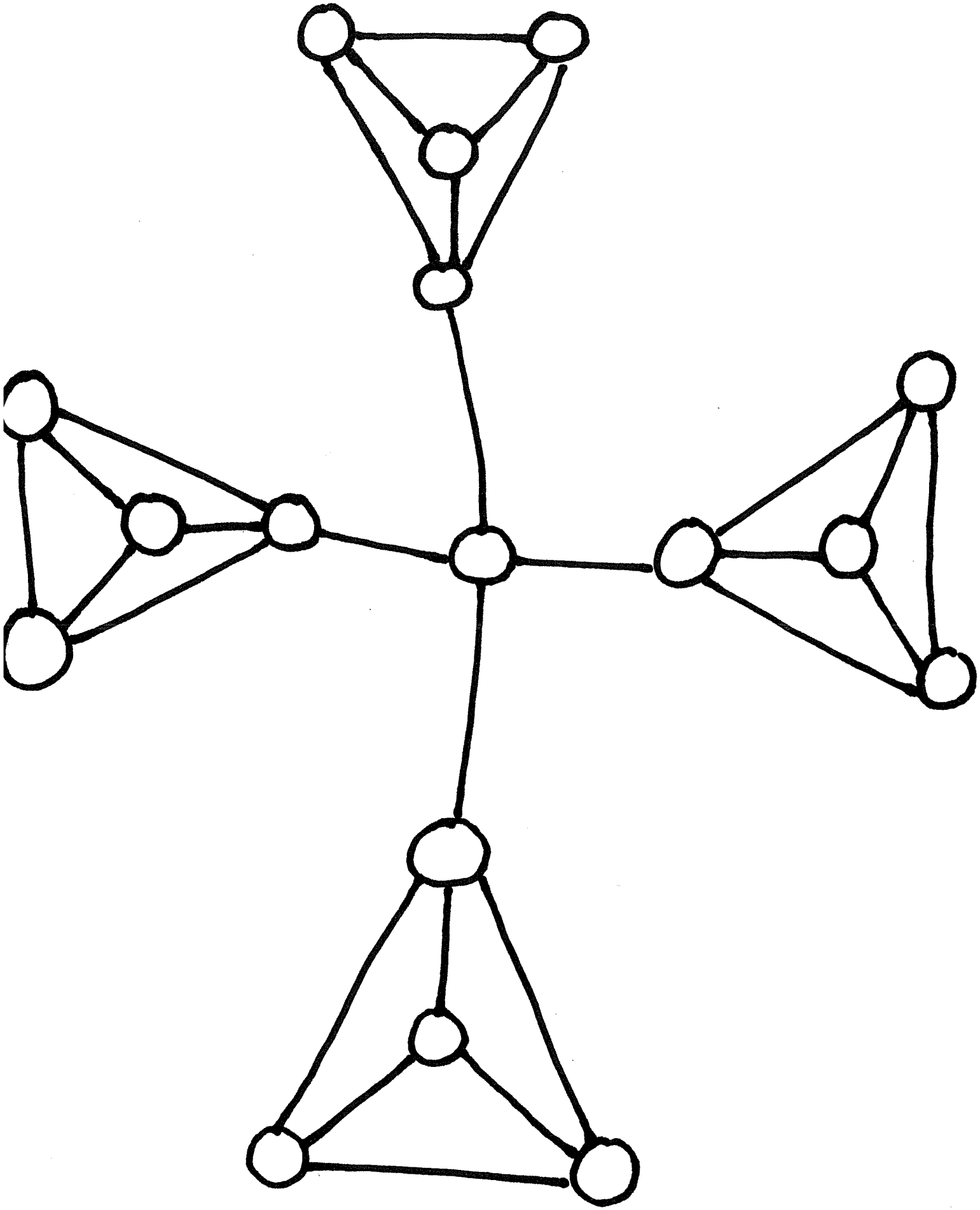

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