Graph Theory - MATH 247

Exam 2

April 29, 2021

Name:
Honor Code Pledge:

Signature:

Directions: Please complete six ~~or~~ of seven problems, including the last problem. Each is worth 10 points. There is a time limit of 2 hours. No notes/texts/calculators/lap-tops/cell-phones/etc. allowed. Please remember to write and sign the Honor Code.

1. Use Dijkstra's algorithm to find the shortest path between the vertices u and v in the graph (Figure 1) below. Show the intermediary calculations that you would have made. Highlight the edges of the path that achieves this shortest path.

2. Give a weighted graph on 8 vertices that contains at least one cycle of length four that demonstrates that the greedy approach to the Traveling Salesman Problem (TSP) can be arbitrarily bad.

3. Determine whether the graph (Figure 2) given below is Hamiltonian. Prove or disprove.

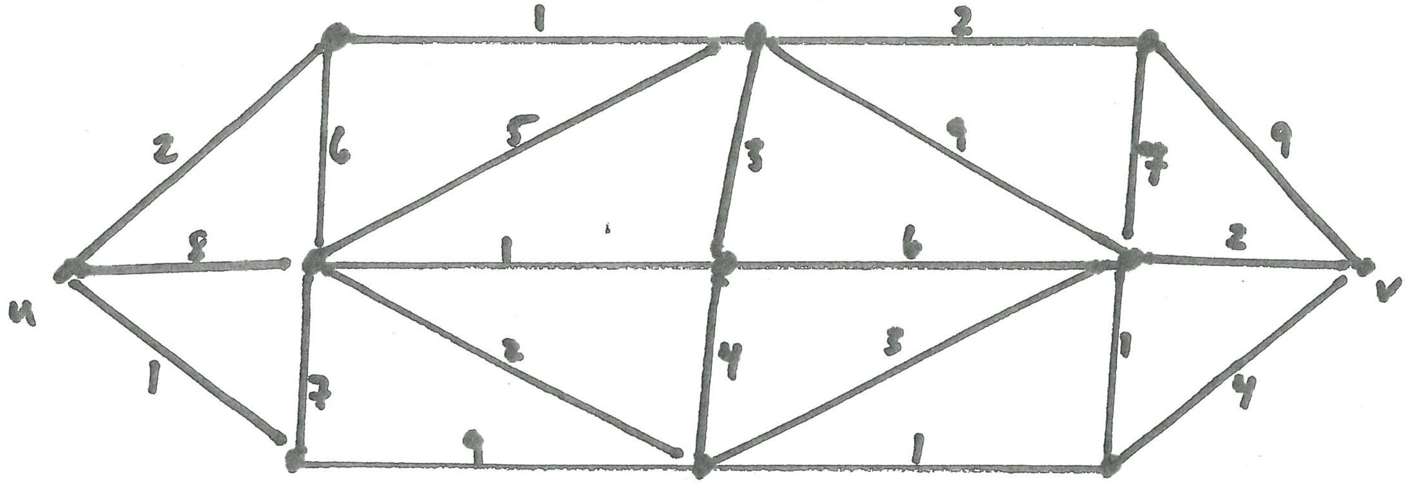
4. State and prove the best-known upper bound for the crossing number of $K_{10,10}$.

5. Prove that the graph C_5 is not graceful. Is C_6 graceful? Prove or disprove.

6. The Petersen graph is a cubic graph that contains a perfect matching. Show that it can be decomposed into edge-disjoint paths of length three. Next show that any cubic graph that satisfies Tutte's condition can also be decomposed into edge-disjoint paths of length three.

7. On a 8×8 chessboard, there are 4 rooks in each row and each column of the board (so a total of 32 rooks). Show that there exist 8 rooks that are pairwise non-attacking. Recall that a rook can only move horizontally or vertically.

Figure 1



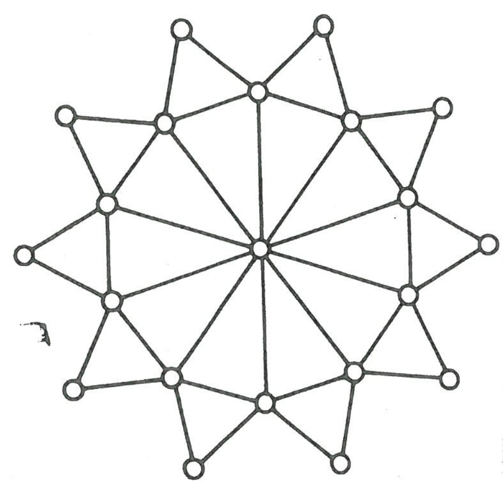


Figure 2

Graph Theory - MATH 247 - Spring 2021

Exam 1

Name:
Honor Code Pledge:

Signature:

Directions: This exam is a closed-book, closed-notes exam. Cell phones should not be used at any time (even to check the time) - please put them away! Please complete **six of seven** problems, including the last problem where you're asked to use induction. Each problem is worth 10 points.

1. **Prüfer codes:** Let $\mathbf{a} = (3, 3, 3, 1, 2, 3, 4)$ be a Prüfer code. Construct the corresponding labelled tree. Also, given the labelled tree below, construct its Prüfer code.

2. **A question based upon Theorem 2.3** Construct/draw a graph of order 24 the degrees of whose vertices are precisely the integers from the following set $\{3, 7, 13, 23\}$.

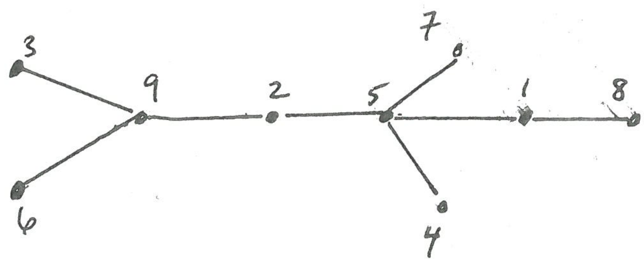
3. **Eulerian graphs**. Prove that every Eulerian bipartite graph has an even number of edges. Next, give a counterexample to show that the following statement is false: every Eulerian simple graph with an even number of vertices has an even number of edges.

4. **Irregularity strength**. Find the irregularity strength of the complete graph on four vertices. Now do the same for the complete graph on five vertices. Give the best upper bound you can think of for the irregularity strength of K_n for $n \geq 3$. Can you prove this upper bound?

5. **Lights out**. In the 3×2 rectangular lattice given below, the only light on is in the upper left corner. Prove that is is impossible to turn off all the lights. (Please be as succinct as possible.)

6. **Degrees of trees** If a tree has exactly one vertex of degree i for $i = 2, 3, \ldots, 10$, then how many leaves does T have? *Justify* your answer.

7. **A proof by induction**. Use induction on the number of vertices n to prove that if G is a tree, then G has at least $\Delta(G)$ leaves. *Hints:* The base case should include $n \leq 3$. For the induction step: delete a leaf and consider two cases. *Requirement:* State the induction hypothesis.



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