Graph Theory - MATH 247 - Spring 2021

Exam 1

Name:
Honor Code Pledge:

Signature:

Directions: This exam is a closed-book, closed-notes exam. Cell phones should not be used at any time (even to check the time) - please put them away! Please complete **six of seven** problems, including the last problem where you're asked to use induction. Each problem is worth 10 points.

1. **Prüfer codes:** Let $\mathbf{a} = (3, 3, 3, 1, 2, 3, 4)$ be a Prüfer code. Construct the corresponding labelled tree. Also, given the labelled tree below, construct its Prüfer code.

2. **A question based upon Theorem 2.3** Construct/draw a graph of order 24 the degrees of whose vertices are precisely the integers from the following set $\{3, 7, 13, 23\}$.

3. **Eulerian graphs**. Prove that every Eulerian bipartite graph has an even number of edges. Next, give a counterexample to show that the following statement is false: every Eulerian simple graph with an even number of vertices has an even number of edges.

4. **Irregularity strength**. Find the irregularity strength of the complete graph on four vertices. Now do the same for the complete graph on five vertices. Give the best upper bound you can think of for the irregularity strength of K_n for $n \geq 3$. Can you prove this upper bound?

5. **Lights out**. In the 3×2 rectangular lattice given below, the only light on is in the upper left corner. Prove that is is impossible to turn off all the lights. (Please be as succinct as possible.)

6. **Degrees of trees** If a tree has exactly one vertex of degree i for $i = 2, 3, \ldots, 10$, then how many leaves does T have? *Justify* your answer.

7. **A proof by induction**. Use induction on the number of vertices n to prove that if G is a tree, then G has at least $\Delta(G)$ leaves. *Hints:* The base case should include $n \leq 3$. For the induction step: delete a leaf and consider two cases. *Requirement:* State the induction hypothesis.