

Graph Theory - MATH 247 - Spring '08

Exam 1

Name:

Honor Code Pledge:

Signature:

Directions: Please complete all but 1 problem. Each problem is worth 10 points. Please complete the problems on separate paper and place them in order upon stapling to this sheet.

1. Prove that if K_n decomposes into triangles, then $n - 1$ or $n - 3$ is divisible by 6.
2. Determine if the following sequences are graphic. If the sequence is not graphic, then indicate why. If the sequence is graphic, give a graph that realizes the sequence.
 - $(5, 5, 5, 4, 4, 4, 3, 3, 2, 2, 1, 1)$
 - $(7, 5, 5, 5, 5, 5, 2, 2)$
3. Let G be a simple graph having no isolated vertex and no induced subgraph with exactly two edges. Prove that G is a complete graph.
4. Prove that every simple graph with at least two vertices has two vertices of equal degree.
5. Determine the number of edges in the k -dimensional hypercube, Q_k , and justify your answer.
6. Draw the de Bruijn graph D_3 and give an Eulerian circuit of this graph.
7. Decompose the Petersen graph into copies of P_4 (where P_4 denotes the path on four vertices).
8. Let P and Q be paths of maximum length in a connected graph G . Prove that P and Q have a common vertex.
9. **Use induction on $n(G)$** to prove that every nontrivial loopless graph G has a bipartite subgraph H such that H has *more than* $e(G)/2$ edges.