Graph Theory - MATH 247 -Fall 2019

Exam 1

Name:

Honor Code Pledge:

Signature:

Directions: This exam is a closed-book, closed-notes exam. Cell phones should not be used at any time (even to check the time) - please put them away! Please complete six of seven problems, including the last problem where you're asked to use induction. Each problem is worth 10 points.

1. Prüfer codes: Let $a = (3, 4, 5, 1, 7)$ be a Prüfer code. Construct the corresponding labelled tree. Also, given the labelled tree below, construct its Prüfer code.

2. Find the irregularity strength of the path on 5 vertices, P_5. Do the same for P_6. Make a conjecture for the irregularity strength of P_n.

3. Prove that if the diameter of a graph G is at least 3, then the complement $\overline{G}$ has domination number at most 2. What property must G have so that $\overline{G}$ has domination number 1?

4. If T is a minimum-weight spanning tree of a weighted graph G, then the u, v-path in T is not necessarily a minimum-weight u, v-path in G. Give an example where it is. Give an example where it is not.

5. The graph below is not bipartite. Determine the fewest number of edges f that need to be deleted to make the graph bipartite. Explain your answer. Is the choice of edges that you made

the only choice with f edges? Why or why not?

6. Let G be an $20-$vertex graph with minimum degree of at least 10. Prove that G is connected. More generally, prove that an $n-$vertex graph with minimum degree at least $\frac{n-1}{2}$ is connected.

7. Prove that every graph with no odd cycles is bipartite using induction on the number of vertices.