Combinatorics - MATH 0345

Exam 3

May 18, 2009

Name:

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Signature

   Directions: Please complete all but one problem.

1. Prove that the addition table of Z_4 is a Latin square without an orthogonal mate.

2. How many $2 - by - n$ Latin rectangles have first row equal to

$$0 \quad 1 \quad 2 \quad \cdots \quad n-1 \quad ?$$

   Estimate how many $3 - by - n$ rectangles have this as their first row.

3. Solve the recurrence relation $h_n = -h_{n-1}+1, \ (n \geq 1); h_0 = 0$, by examining the first few values for a formula and then proving your conjectured formula by induction.

4. Call a subset S of the integers $\{1, 2, \ldots, n\}$ *extraordinary* provided its smallest integer equals its size:
$$\min\{x : x \in S\} = |S|.$$

   For example, $S = \{3, 7, 8\}$ is extraordinary. Let g_n be the number of extraordinary subsets of $\{1, 2, \ldots, n\}$. Determine a linear recurrence relation for g_n.

5. Let $p_1 \leq p_2 \leq \ldots \leq p_k$ be primes and $n = p_1 \times p_2 \times \ldots \times p_k$. Prove that $N(n)$, the largest number of MOLS of order n, is at least $p_1 - 1$.

6. Solve the recurrence relation $h_n = 8h_{n-1} - 16h_{n-2}, \ (n \geq 2)$ with initial value $h_0 = -1$ and $h_1 = 0$.