Combinatorics - MATH 0345

Exam 3

December 7, 2010

Name:
Honor Code Pledge

Signature

Directions: Please complete each problem on the first page, and two of the three on the second page.

1. A Latin Square A based on $\mathbf{Z}_n$ of order n is *symmetric*, provided the entry a_{ij} at row i, column j equals the entry a_{ji} at column j, row i for all $i \neq j$. It is *idempotent* provided that its entries on the diagonal running from upper left to lower right are $0, 1, 2, \ldots, n-1$. Prove that a symmetric, idempotent Latin square has odd order and construct an example of order 5.

2. Determine the corner-symmetry group of a rectangle that is not a square. Use this and Burnside's Lemma to determine the number of nonequivalent colorings with the colors red and blue. Do the same with p colors.

3. One may give an upper bound on the number 3×3 games of tic-tac-toe as follows: for each of the 9 positions we have two possible choices, a cross (an 'X') and a naught (an 'O'), thus a total of $2^9 = 512$ possible games. Unfortunately, this allows for a game ending with all 9 positions filled with a cross and games for which a rational player would have stopped as a winner must have been determined before the ninth move. However, 512 still serves as an upper bound for the number of games. With respect to this set, determine the number of non-equivalent games.[1]

[1] If you reason to impress me, can you achieve a better upper bound?

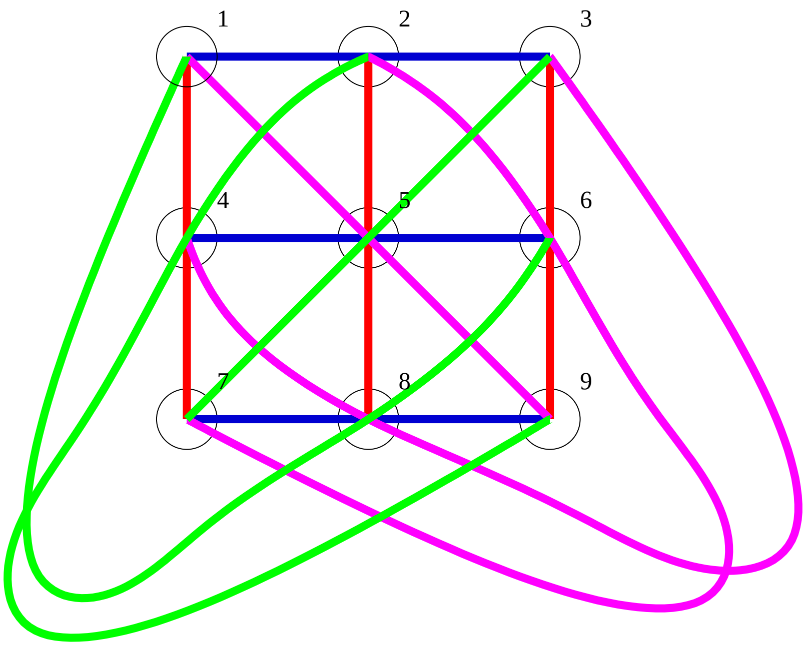

Figure 1: A resolvable Steiner Triple System

4. (Note that there are four distinct colors in the figure below.) In Figure 1 is an illustration of a specific instance of a resolvable Steiner Triple System. Please explicitly give the complementary design and determine each of the parameters (v, k, r, b, λ) in this complementary design.

5. In a BIBD with blocks of size 5, on a point set of size 11, and where each pair of points occurs together exactly twice, determine the replication number of each point and the number of blocks in the design.

6. Again consider the resolvable Steiner Triple System illustrated in Figure 1. Modify the design in the following way: for each color class add a new point and extend each block in the color class to include this new point. The obtained set system is not a BIBD, can you remedy this and simultaneously give the parameters (v, k, r, b, λ) of this BIBD?