Combinatorics - MATH 0345

Exam 3

December 18, 2006

Name:
Honor Code Pledge

Signature
Directions: Please complete all but 1 problem. If you complete all seven problems, I will count your best six.

1. Solve the following recurrence relation by examining the first few values for a formula and then proving your conjectured formula by induction.

$$h_n = 2h_{n-1} + 1, \quad (n \geq 1); h_0 = 1$$

2. Solve the nonhomogeneous recurrence relation

$$\begin{aligned} h_n &= 4h_{n-1} + 4^n, \quad (n \geq 1) \\ h_0 &= 3 \end{aligned}$$

3. Construct 2 mutually orthogonal latin squares (MOLS) of order 9.

4. A Latin Square A based on $\mathbf{Z}_n$ of order n is *symmetric*, provided the entry a_{ij} at row i, column j equals the entry a_{ji} at column j, row i for all $i \neq j$. It is *idempotent* provided that its entries on the diagonal running from upper left to lower right are $0, 1, 2, \ldots, n-1$. Prove that a symmetric, idempotent Latin square has odd order and construct an example of order 5.

5.
   - Does there exist a BIBD with parameters $b = 10, v = 8, r = 5$ and $k = 4$? Justify your answer.
   - True or false: In a BIBD, if $k = 3$, then either λ is even or v is odd. (Prove or give a counterexample.)

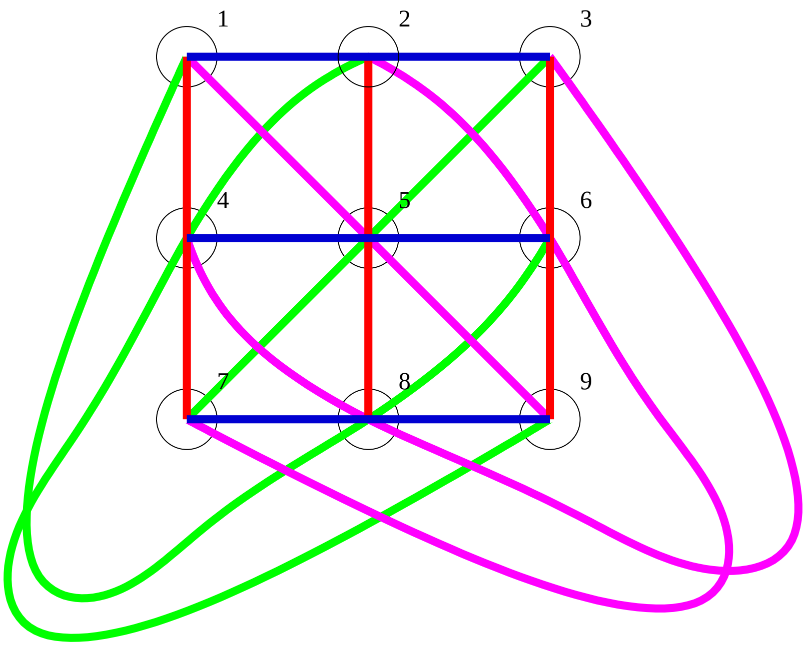

Figure 1: What am I?

6. The figure in Figure 1 recently appeared on a bathroom wall in Warner Hall. What is it? Be as specific as possible.

7. Suppose that we have an unlimited pile of nickels, dimes and quarters and wish to make change for a dollar. How many ways are there to do this? What if the question asked for the number of ways to make change for two dollars, three dollars, a hundred dollars? (You do not need to compute this directly, but rather give an efficient way one could express all the answers at the same time.)