Combinatorics - MATH 0345

Exam 2

April 19, 2018

Name:
Honor Code Pledge:

Signature:
Directions: Please complete all of the problems.

1. Determine the number of integral solutions of the equation

$$x_1 + x_2 + x_3 + x_4 = 20$$

that satisfy

$$1 \leq x_1 \leq 6, \quad 0 \leq x_2 \leq 7, \quad 4 \leq x_3 \leq 8, \quad 2 \leq x_4 \leq 6.$$

2. Solve the nonhomogeneous recurrence relation $h_n = 2h_{n-1} + n, (n \geq 1)$ and $h_0 = 1$.

3. Solve the recurrence relation by using the method of generating functions: $h_n = h_{n-1} + h_{n-2}(n \geq 2)$ and $h_0 = 1, h_1 = 3$.

4. Consider the integers in the set $\{1, 2, 3, 4, 5\}$ and we wish to count permutations of these in which we never see three consecutive numbers in increasing order. For instance, the permutation $5, 1, 2, 4, 3$ would be bad because we see 1 immediately followed by 2, which is immediately followed by 4. However, the permutation $1, 5, 2, 4, 3$ is good since for any three consecutive numbers we have at least one decrease in value. Use the Principle of Inclusion Exclusion to count the number of such permutations.

5. Use induction to prove that the Fibonacci number f_{5k} is a multiple of 5, for all integers $k \geq 0$. That is, prove by induction that every fifth Fibonacci number is divisible by 5. *Hint:* in the induction step use the Fibonacci recurrence more than once!

6. Let $(X, \leq)$ be a finite poset. Consider the set A of maximal elements. (Recall that an element b of the poset is said to be maximal such that no element y satisfies $b < y$.) [3 sentences maximum] Prove that the set A is an antichain. [4 sentences maximum]

Now prove that A is a maximal antichain. That is, prove that there is no antichain larger than A that properly contains A. Please indicate how you use the hypothesis of *finite*.