Combinatorics - MATH 0345

Exam 2

November 15, 2012

Name:
Honor Code Pledge:

Signature:
Directions: Please complete all but one of the problems. Each problem is worth 10 points – elegance counts.

1. Find one binomial coefficient equal to the following expression (and justify your claim):

$$\binom{n}{k} + 3\binom{n}{k-1} + 3\binom{n}{k-2} + \binom{n}{k-3}.$$

2. Use Newton's binomial theorem to approximate $\sqrt{40}$.

3. Solve the following linear homogeneous recurrence relation:

$$h_n = 3h_{n-1} - 2h_{n-2}, \quad h_0 = 2, h_1 = 2.$$

4. Determine the number of integral solutions of the equation

$$x_1 + x_2 + x_3 + x_4 = 15$$

   that satisfy $1 \le x_1 \le 5, 0 \le x_2 \le 6, 2 \le x_3 \le 7, 0 \le x_4 \le 15$.

5. In a partition of the subsets of $\{1, \ldots, n\}$ into symmetric chains, how many chains have only one subset in them? two subsets? k subsets? (In a *symmetric chain partition* each subset in a chain has one more element than the subset that precedes it in the chain, and the size of the first subset in a chain plus the size of the last subset in the chain equals n. (If the chain contains only one subset, then it is both first and last, so twice its size is n; that is, its size is $n/2$ and n is even.))

6. A function from a set A *onto* a set B is defined as a function from A to B such that each member of B is associated with at least one member of A. Using the Principle of Inclusion-Exclusion show that the number of functions from an m-element set onto an n-element set is equal to

$$\sum_{i=0}^{n}(-1)^i\binom{n}{i}(n-i)^m.$$

7. **Theorem** *Let $(X, \leq)$ be a finite partially ordered set, and let r be the largest size of a chain. Then X can be partitioned into r but no fewer antichains.* Use this theorem to show that, if m and n are positive integers, then a partially ordered set of $mn+1$ elements has a chain of size $m+1$ or an antichain of size $n+1$.