Name: 
Honor Code Pledge

Signature

Directions: Please complete all but 1 problem. If you complete all six problems, I will count your best five.

1. Let \( n \) and \( k \) be positive integers. Give a combinatorial proof that

\[
\sum_{k=1}^{n} k \binom{n}{k}^2 = n \binom{2n-1}{n-1}.
\]

2. Given that

\[
\binom{1/2}{k} = \frac{(-1)^{k-1}}{k^{2k-2}} \binom{2k-2}{k-1},
\]

use Newton’s Binomial Theorem to approximate \( \sqrt{40} \). (You may leave your answer as a sum.)

3. An opinion poll reports that the percentage of voters who would be satisfied with each of three candidates \( A, B, C \) for President is 65\%, 57\%, 58\% respectively. Further, 28\% would accept \( A \) or \( B \), 30\% \( A \) or \( C \), 27\% \( B \) or \( C \) and 12\% would be content with any of the three. What do you conclude?

4. What is the number of ways to place six nonattacking rooks on the 6-by-6 boards with forbidden positions as shown? (See opposite side. You may leave your answer as a sum.)

5. Prove that the \( n \)th Fibonacci number \( f_n \) is the integer that is closest to the number

\[
\frac{1}{\sqrt{5}} \left( 1 + \frac{\sqrt{5}}{2} \right)^n.
\]

6. There are \( n \) seating positions arranged in a line. Prove that the number of ways of choosing a subset of these positions, with no two chosen positions consecutive, is \( f_{n+1} \) (the \((n + 1)\)th Fibonacci number).
• If the $n$ positions are arranged around a circle, show that the number of choices is $f_n + f_{n-2}$ for $n \geq 2$. 