Combinatorics - MATH 0345

Exam 2

November 17, 2006

Name:
Honor Code Pledge

Signature
Directions: Please complete all but 1 problem. If you complete all six problems, I will count your best five.

1. Let n and k be positive integers. Give a combinatorial proof that

$$\Sigma_{k=1}^{n} k \binom{n}{k}^2 = n\binom{2n-1}{n-1}.$$

2. Given that $\binom{1/2}{k} = \frac{(-1)^{k-1}}{k2^{2k-1}}\binom{2k-2}{k-1}$, use Newton's Binomial Theorem to approximate $\sqrt{40}$. (You may leave your answer as a sum.)

3. An opinion poll reports that the percentage of voters who would be satisfied with each of three candidates A, B, C for President is $65\%, 57\%, 58\%$ respectively. Further, 28% would accept A or B, 30% A or C, 27% B or C and 12% would be content with any of the three. What do you conclude?

4. What is the number of ways to place six nonattacking rooks on the 6-by-6 boards with forbidden positions as shown? (See opposite side. You may leave your answer as a sum.)

5. Prove that the nth Fibonacci number f_n is the integer that is closest to the number

$$\frac{1}{\sqrt{5}}(\frac{1+\sqrt{5}}{2})^n.$$

6.
   - There are n seating positions arranged in a line. Prove that the number of ways of choosing a subset of these positions, with no two chosen positions consecutive, is f_{n+1} (the $(n+1)^{th}$ fibonacci number).

- If the n positions are arranged around a circle, show that the number of choices is $f_n + f_{n-2}$ for $n \geq 2$.