

Combinatorics - MATH 0345

Exam 1

March 12, 2015

Name:

Honor Code Pledge:

Signature:

Directions: Please complete **six of the seven** problems. Justify all solutions — partial work receives partial credit. Each problem is worth ten points. Answers may be left in terms of factorials, binomial coefficients, products and sums of these, and the like. There is a time limit of 2 hours.

1. A game is played between two players, alternating turns as follows: The game starts with an empty pile. When it is her turn, a player may add either 1, 2, 3 or 4 coins to the pile. The person who adds the 100th coin to the pile is the winner. Determine whether it is the first or second player who can guarantee a win in this game. What is the winning strategy?
2. Consider the multi-set $\{n \cdot a, 1, 2, 3, \dots, n\}$ of size $2n$. Determine the number of its n -combinations.
3. A hand in a game of bridge consists of 13 of the 52 cards in a standard deck. How many bridge hand are there? How many bridge hands have exactly two aces? How many bridge hand have at least one heart? How many bridge hands have four cards in one suit and three cards in each of the other three suits?
4. There are eight guests at a party and they sit around an octagonal table with one guest at each edge. If each place at the table is marked with a different person's name and initially everybody is sitting in the wrong place, prove that the table can be rotated in such a way that at least 2 people are sitting in the correct place.
5. Prove the following identity: $3^n = \binom{n}{0}2^0 + \binom{n}{1}2^1 + \dots + \binom{n}{n}2^n$.
6. Middlebury College students belong to one of five commons: Atwater, Brainerd, Cook, Ross and Wonnacott. If I were to survey 19 such students, then how many different ways could the survey turn out? What if from among the 19 selected, I recognized someone I knew who belonged to Brainerd?

7. Knowing that $r(3, 3) = 6$, $r(3, 4) = 9$, $r(3, 5) = 14$ and $r(4, 4) = 18$ give and prove an upper bound on $r(4, 5)$. (Please don't try to show that what you've obtained is sharp. And you needn't use all of the facts listed.)