

Combinatorics - MATH 0345

Exam 1

March 12, 2009

Name:

Honor Code Pledge:

Signature:

Directions: Please complete ALL the problems. There is a time limit of $25 \cdot r(3, 3)$ minutes.

1. Consider an n -by- n board and L -tetrominoes (4 squares joined in the shape of an L). Show that if there is a perfect cover of the n -by- n board with L -tetrominoes, then n is divisible by 4. What about m -by- n boards?
2. Prove that the number of permutations of m Apes and at most n Bananas equals

$$\binom{m+n+1}{m+1}$$

3. (a) Consider the multiset $\{n \cdot a, 1, 2, 3, \dots, n\}$ of size $2n$. Determine the number of its n -combinations.
(b) Consider the multiset $\{n \cdot a, n \cdot b, 1, 2, 3, \dots, n+1\}$ of size $3n+1$. Determine the number of its n -combinations.
4. A bag contains 50 gold coins, 50 silver coins, and 50 copper coins. If I remove one coin out of the bag every minute, how long will it be before I am assured of having picked at least 5 coins of the same kind?
5. Let q be a positive integer with $q \geq 3$. Determine the Ramsey number $r_3(3, 3, q)$.
6. Use the binomial theorem to prove that

$$2^n = \sum_{k=0}^n (-1)^k \binom{n}{k} 3^{n-k}.$$

7. Use a combinatorial argument to prove that for all positive integers m_1, m_2 , and n ,

$$\sum_{k=0}^n \binom{m_1}{k} \binom{m_2}{n-k} = \binom{m_1 + m_2}{n}.$$

8. A college dormitory has 250 students. For every pair of students, A and B , there is a language that A speaks that B does not, and a language that B speaks that A does not. What is the smallest total number of languages that could be known by the students?