Combinatorics - MATH 0345

Exam 1

October 11, 2012

Name:
Honor Code Pledge:

Signature:

Directions: Please complete five of the first 6 problems, plus the last one. Justify all solutions — partial work receives partial credit. Each of the problems is worth ten points. There is a time limit of $25 \cdot r(3,3)$ minutes or so.

1. Give a combinatorial proof of the following identity,
$$k\binom{n}{k} = n\binom{n-1}{k-1},$$
where n and k are positive integers.

2. There are two identical circular tables, each of which sits 9. The 18 of us are to sit down at them, but Molly refuses to sit next to me. How many ways are there to make a seating arrangement that honors Molly's preference?

3. A man works in a building located nine blocks east and eight blocks north of his home. Every day he walks 17 blocks to work.
   (a) How many different routes are possible for him?
   (b) How many different routes are possible if the one block in the easterly direction, which begins four blocks east and three blocks north of his home, is under water (and he can't swim)? (Hint: Count the routes that use the block under water.)

4. What is the number of nondecreasing sequences of length 4 whose terms are taken from $1, 2, \ldots, 10$?

5. A group of 24 people are to be arranged into 3 teams each with 8 players.
   (a) Determine the number of ways if each team has a different name.

(b) Determine the number of ways if the teams don't have names.

6. Determine the coefficient on the monomial x^5y^2 upon expanding $(2x-3y)^7$. Now do the same for the monomial x^3y^3 – no, I didn't make a typo.

7. Given 5 lattice points p_1, p_2, p_3, p_4, p_5 in the plane (i.e., points of the form (x, y) where both x and y are integers), use the pigeonhole principle to show that there is a pair where the midpoint of the line segment joining them is also a lattice point. **Hint:** Define a function $f : \{p_1, \ldots, p_5\} \to \{(0,0), (0,1), (1,0), (1,1)\}$ by mapping p_i to the ordered pair (a_i, b_i), where $a_i \equiv x_i \mod 2$ and $b_i \equiv y_i \mod 2$.