Combinatorics - MATH 0345

Exam 1

October 7, 2010

Name:
Honor Code Pledge:

Signature:

Directions: Please complete ALL the problems. Justify all solutions — partial work receive partial credit. Each of the eight problems is worth ten points. There is a time limit of $25 \cdot r(3,3)$ minutes or so.

1. Can the following partial square be completed to obtain a magic square of order 4?

$$\begin{bmatrix} 2 & 3 & & \\ 4 & & & \\ & & & \\ & & & \end{bmatrix}$$

2. How many permutations are there of the letters of the word MIDDLEBURY? How many 4-permutations are there of these 10 letters?

3. How many integral solutions of

$$x_1 + x_2 + x_3 + x_4 = 30$$

satisfy $x_1 \geq 2, x_2 \geq 0, x_3 \geq -5$, and $x_4 \geq 8$?

4. Prove that, for any $n+1$ integers $a_1, a_2, \ldots, a_{n+1}$, there exist two of the integers a_i and a_j with $i \neq j$ such that $a_i - a_j$ is divisible by n.

5. A lattice point is a point in a coordinate plane for which both coordinates are integers. Prove that no matter what five lattice points might be chosen in the plane at least one of the segments that joins two of the chosen points must pass through some lattice point in the plane. (Hint: consider the "parity" classes for the coordinates for the five lattice points.)

6. Use the binomial theorem to expand $(y + 2z)^6$.

7. Use a combinatorial argument to prove that for all positive integers $r \geq m \geq k$,

$$\binom{r}{m}\binom{m}{k} = \binom{r}{k}\binom{r-k}{m-k}.$$

8. What does it mean when we write $r(5, 5) \leq 49$? What does it mean when we write $r(5, 5) \geq 43$?