Combinatorics - MATH 0345

Exam 1

October 13, 2006

Name:
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Signature
Directions: Please complete all but 1 problem. There is a time limit of 2 hours.

1. Show that a magic square of order 3 must have a 5 in the middle position. Deduce that there are exactly 8 magic squares of order 3.

2. Prove that in a simple graph with n vertices, with $n > 1$, that there exist at least two vertices of the same degree.

3. Prove that $r(\overbrace{3, 3, \ldots 3}^{k+1}) \leq (k+1)(r(\overbrace{3, 3, \ldots 3}^{k}) - 1) + 2$.

4. A football team of 11 players is to be selected from a set of 15 players, 5 of whom can play only in the backfield, 8 of whom can play only on the line, and 2 of whom can play either in the backfield or on the line. Assuming a football team has 7 men on the line and 4 in the backfield, determine the number of football teams possible.

5. Twenty different books are to be put on five book shelves, each of which holds at least twenty books.

   - How many different arrangements are there if you only care about the number of books on the shelves (and not which book is where)?
   - How many different arrangements are there if you care about which books are where but the order of the books on the shelves doesn't matter?
   - How many different arrangements are there if the order on the shelves does matter?

6. What is the number of integral solutions of the equation:

$$z_1 + z_2 + z_3 = 11$$

if $z_1 \geq 2$ and $z_2 > z_3$.

7. Use *combinatorial* reasoning to prove the identity (in the form given).

$$\binom{n}{k} - \binom{n-3}{k} = \binom{n-1}{k-1} + \binom{n-2}{k-1} + \binom{n-3}{k-1}.$$

   (Hint: Let S be a set with three distinguished elements a, b and x and count certain k-combinations of S.)

8. Expand $(\frac{1}{2}z + 4x)^5$ using the binomial theorem.

9. There are two circular tables, each of which sit four. Given 7 people how many ways are there to make a seating arrangement for these 7 people.