

# Math 345: Combinatorics

## Spring Term 2024

### Course Description

February 12, 2024

**Instructor:** John Schmitt

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My (and course) **webpage:** <http://community.middlebury.edu/~jschmitt/>

**Office Hours:** Tuesday 1–2:15pm, Wednesday 9–10:30am, Friday 9:30–11am, by arrangement and the 15 minutes that immediately follows class

**Meeting times:**

Section A, MWF, 1:10pm–2:00pm, Warner Science Hall 104

**Overview:**

The study of Combinatorics involves the following. Given a finite set of objects and a set of rules placed upon these objects, we ask the following questions:

1. Does there exist an arrangement of the objects that satisfy the rules? This is the question of **existence**.
2. If such an arrangement exists, how many are there? This is the question of **enumeration**.
3. If such an arrangement exists, can one construct such an arrangement? This is the question of **construction**.
4. Can one construct an arrangement that optimizes some particular parameter? This is the question of **optimization**.

Our focus mostly will be on aspects of enumeration, though we will spend some time with the existence question via design theory. I hope to highlight my favorite area: extremal

combinatorics, which deals with understanding the structure of combinatorial objects satisfying some given property and that are as ‘large’ or ‘small’ as possible. Questions from graph theory are considered in MATH 0247.

**Textbook:** *Introductory Combinatorics, Fifth Edition* by Richard Brualdi, Prentice Hall, NJ, 2018(re-issue). A list of errata is maintained at the author’s webpage, <https://www.math.wisc.edu/~brualdi/>.

### Supplemental texts available in Davis Family Library

- *A Course in Combinatorics*, Van Lint and Wilson
- *Enumerative Combinatorics*, R. Stanley
- *Introductory Combinatorics*, K. Bogart
- There are many others.

**Homework:** Homework will be assigned on a weekly basis. The content of this course is best learned by *practicing problems*. I **encourage you to work together**. However, the write-up of homework solutions *should be done on your own*. Please see the accompanying “thoughts” on homework.

Students who violate the Honor Code by copying solutions found via the internet, a solutions manual or some other source will *at minimum* forfeit the entire portion of the homework grade. The set of homework problems is considered as one assignment, with due dates spread over the course of the semester. There have been occasions in the past when a student has violate the Honor Code and failed the course; please, let’s not repeat this!

Also, beginning with the second homework assignment, you will need to have at least one problem from each assignment typeset using the mathematical typesetting tool  $\text{\LaTeX}$ . In asking this of you I assume that as you are enrolled in this course, you have a future in the mathematical sciences. This tool is indispensable for such a future. Most people now use  $\text{\LaTeX}$  via the on-line, collaborative tool known as Overleaf (see <https://www.overleaf.com/home-2>). For older folks:  $\text{\LaTeX}$  is available for the Mac via TeXShop, available for free download here, <http://pages.uoregon.edu/koch/texshop/>. For those with a PC, MikTeX and TexnicCenter are available from the following locations <http://miktex.org/2.9/setup> and <http://www.texniccenter.org/resources/downloads/29>.

**Special Needs:** If you require special arrangements for class or during tests/exams please communicate with me as soon as possible to make such arrangements. Also, please inform me if you are color-blind as the use of colors can be an important part of a lecture.

**Grading percentages:**

Homework	25
Midterm 1	25
Midterm 2	25
Midterm 3 (Final)	25

**Assignment of grades:**

The assignment of grades will follow the scheme below *at a minimum*. Plus and minus will be determined at the end of the semester.

90 and above	A
80 - 89	B
70 - 79	C
60 -69	D
below 60	F

**Midterm exam schedule:**

Thursday evening, March 14, 7:30–9:30pm

Thursday evening, April 18, 7:30–9:30pm

**Final exam:**

Thursday, May 16th, 2–5pm. (This time-slot is the “combined exam” time-slot, not the originally assigned Friday night slot.)

**Open problems:**

Mathematics is a living subject. According to the celebrated mathematician Paul Erdős, the point of life is “to prove and conjecture.” Therefore, anyone solving an unsolved problem in combinatorics (including graph theory) will automatically receive an A for the course. The problem must appear in a peer-reviewed mathematics publication (or approved website) and you must be “current” with all your work.

Approved websites include:

- <http://garden.irmacs.sfu.ca>
- <http://www.math.uiuc.edu/~west/openp/>
- [http://www.dmoz.org/Science/Math/Combinatorics/Graph\\_Theory/Open\\_Problems/](http://www.dmoz.org/Science/Math/Combinatorics/Graph_Theory/Open_Problems/)

**Our textbook author:** The author of the text, Richard Brualdi, has a website <http://www.math.wisc.edu/~brualdi/>, with a link for info and updates on our text.

**Absences:** Please see me as far in advance as possible for absences that will occur on the day of an exam. Any such absences, or unforeseen ones, must be documented in writing by the appropriate person.

**Honor Code:** The Honor Code will be observed throughout this class and for all examinations. Exams will be “closed notes, closed books,” unless otherwise noted. If you have a question about how the Honor Code applies to this class, please ask. In particular, failure to comply with the homework policy (see above) will be considered a violation of the Honor Code. The use of generative AI is forbidden for all problem sets and exams.

## Outline of Topics

1. Introductory Problems  
Chessboard covers, magic squares, Euler's 36 officers, and others
2. Permutations and Combinations  
Basic counting principles
3. The Pigeonhole Principle  
Simple and strong form, and Ramsey's Theorem
4. Generating Permutations and Combinations
5. The Binomial Coefficients  
Pascal's formula, the binomial theorem
6. Principle of Inclusion-Exclusion
7. Recurrence Relations and Generating Functions
8. Special Counting Sequences  
Catalan and Stirling numbers
9. Combinatorial Designs  
Block designs, triple systems and latin squares
10. Pólya Counting (if time permits)  
Burnside's lemma, Pólya's counting formula

## Goals of the Course

- gain an understanding of the fundamental concepts of combinatorics,
- gain an understanding of how counting arguments may be used within mathematics,
- develop the ability to write a logical and coherent proof,
- introduce topics suitable for a senior thesis,
- develop a desire for further study in related areas, including graph theory and computer science,
- gain an appreciation for combinatorial applications to the physical, biological and social sciences, etc.,
- appreciate the beauty and sophistication of combinatorial arguments,
- have fun!