MATH 345 - Spring '24

Problem Sets

Assignments are mainly taken from our class text, *Introductory Combinatorics, 5th edition* by Richard Brualdi.

A few comments about the grading of problem sets: Each problem from each problem set will be worth 10 points, unless otherwise noted. Partial credit is awarded and I hope to see progress over the course of the semester.

Problems marked as "Also do" are not collected. Rather these serve as a collection of problems from which I often choose a couple (or variations of them) for a mid-term. You should collaborate on these with each other.

1. Due Friday, February 16 (week 1)
   Read: Chapters 1 and 2, the syllabus and *Thoughts on Homework*
   Turn in: Pages 60—68 numbers 19, 21, 54 (read 53 for a further understanding of what a tower is; back of text carries a hint)
   Also do: 2,7, 28, 29, 39, 45, 51

2. Due Friday, February 23 (week 2)
   Note: For this problem set and all subsequent ones, it is required that you typeset at least one problem.
   Hint: When using the pigeonhole principle, it is good to write phrases like, "regardless of the choice, we have ..." It is good to clearly identify the pigeonholes and the pigeons. Please avoid being prescriptive about how a choice is made – this is often an error.
   Read: Chapter 3.1, 3.2, 3.3 Martin Gardner's article *The Power of the Pigeonhole* and *Ramsey Theory*
   Turn in:

   - Pages 60–68 number 38
   - Pages 82—85 numbers 4, 23
   - Show that the theorem of Erdős and Szekeres is "sharp" for $n = 4$. That is, show that you can arrange the first 16 positive integers in such a way so that there is neither an increasing subsequence of length 5 nor a decreasing subsequence of length 5. Can you generalize your arrangement for all n? (N.B. It is insufficient

to give an arrangement but not give an argument as to why the arrangement works.)

Also do: Pages 82–85 number 5, 15, 18, 22, 25, 28

3. Due Friday, March 1 (week 3)
   Turn in:
   - Pages 82-85 numbers 27 (see next problem, too)
   - In Problem 27 given above, it is incorrect to assume that the collection of subsets has an element common to all of the subsets. For $n = 5$ give (i.e. *construct*) a collection of $2^{5-1} = 16$ subsets $S_1, \ldots, S_{16}$ of the set $\{1, \ldots, 5\}$ such that $S_i \cap S_j \neq \emptyset$ and the intersection of all of them (i.e. $\cap_{i=1}^{16} S_i$) is empty. Can you generalize your construction to other values of n? (N.B. It is insufficient to give an arrangement but not give an argument as to why the arrangement works.)
   - We skip Chapter 4 of the text.
   - Read Chapter 5
   - Pages 153 –160 numbers 16, 17, 25

     Also do Pages 153–160 numbers 13, 15, 22, 28, 35, 40, 47. Note that in this section there are numerous typos, including in problems 26, 44 and 45. I haven't assigned these, but you can find the typos in the errata sheet posted on the course webpage or you can try finding them yourself!
4. Due Friday, March 8 (week 4)
   Turn in: Pages 153–160 numbers 30, 36, 37 and 46
5. EXAM - Thursday evening, March 14 (π-day)
6. Due Friday, March 29: Pages 200-205 - number 3, 7, 9, 16, 24(a) (week 6)

   Also do - 2, 10, 11, 12, 15, 16, 17, 21, 22, 23 (for last identity, can you give a combinatorial proof as well?), 26, 29, 31
7. Due Friday, April 5: Pages 257–264 1(d), 8, 14 (b), 48 (c) (week 7)

   Also do 1(a), 2, 11, 34, 43, 45, 9, 14(a), 14(c), 16, 27, 38(c), 48(b)
8. Week 8 - TBA
9. Exam (week 9) - Thursday, April 18

10. Due Friday, April 26 (week 10): Prove that Fisher's Inequality holds in the case that $\lambda = 1$ without using linear algebra. (Hint: Consider a point x and a block B not containing x and recall the established arithmetic conditions that we have.). Pages 388 – 394 numbers 20, 28, 33,

    Also do: Pages 388 – 394 numbers 20, 21, 28, 31, 32, 33, 35

11. Due Friday, May 3 (week 11): Pages 388 – 394 numbers 42, 48, 55, 60.
    Read Martin Gardner's *Euler's Spoilers*

    Also do: Pages 388 – 394 numbers 44, 52, 57, 58, 59, 61, 62

12. Due Friday, May 10 (week 12): Polyá Counting! – Pages 576–581 numbers 10 and 22

    Some help on number 10: The symmetry group is the set of actions on the geometric object that we can make. For instance, the 180-degree rotation, a flip over a horizontal line. These are the elements of that group. Once we label the corners of the object, then each of these actions gives rise to a permutation of these labels. These permutations are the elements of the corner symmetry group. Notice that the permutation of the corners associated with, say, a horizontal flip of the square might interchange corners 1 and 4 and interchange corners 2 and 3. So, you could identify this permutation as: $1 \to 4, 2 \to 3, 3 \to 2, 4 \to 1$ (or preferably using the two-line notation as the text does).

    To make the distinction between a symmetry group and corner symmetry group is perhaps a bit mysterious. Why bother doing this? This is perhaps only answered if we bring into the conversation the edge symmetry group. So, go ahead and label the edges of the rectangle and then perform the same action (as we did above) of a flip over the horizontal. That same action will keep two of the sides fixed, which is necessarily a different permutation than the permutation given above.

    Also do Pages 576–581 numbers 8, 11, 15, 18, 23(a)

13. Final Exam: Friday, May 16, 2–5pm in our Warner 104 classroom