MATH 345 - Fall '21

Problem Sets

Assignments are mainly taken from our class text, *Introductory Combinatorics, 5th edition* by Richard Brualdi.

A few comments about the grading of problem sets: Each problem from each problem set will be worth 10 points, unless otherwise noted. Partial credit is awarded and I hope to see progress over the course of the semester.

Problems marked as "Also do" are not collected. Rather these serve as a collection of problems from which I often choose a couple (or variations of them) for a mid-term. You should collaborate on these with each other.

1. Due Friday, September 17
   Read: Chapters 1 and 2, the syllabus and *Thoughts on Homework*
   Turn in: Pages 60—68 numbers 19, 21, 54 (read 53 for a further understanding of what a tower is; back of text carries a hint)
   Also do: 2,7, 28, 29, 39, 45, 51

2. Due Friday, September 24
   Note: For this problem set and all subsequent ones, it is required that you typeset at least one problem.
   Hint: When using the pigeonhole principle, it is good to write phrases like, "regardless of the choice, we have ..." It is good to clearly identify the pigeonholes and the pigeons. Please avoid being prescriptive about how a choice is made – this is often an error.
   Read: Chapter 3.1, 3.2, 3.3 Martin Gardner's article *The Power of the Pigeonhole* and *Ramsey Theory*
   Turn in:

   - Pages 60–68 number 38
   - Pages 82—85 numbers 4, 23
   - Show that the theorem of Erdős and Szekeres is "sharp" for $n = 4$. That is, show that you can arrange the first 16 positive integers in such a way so that there is neither an increasing subsequence of length 5 nor a decreasing subsequence of length 5. Can you generalize your arrangement for all n? (N.B. It is insufficient

to give an arrangement but not give an argument as to why the arrangement works.)

Also do: Pages 82–85 number 5, 15, 18, 22, 25, 28

3. Due Friday, October 1
   Turn in:
   - Pages 82-85 numbers 27 (see next problem, too)
   - In Problem 27 given above, it is incorrect to assume that the collection of subsets has an element common to all of the subsets. For $n = 5$ give (i.e. *construct*) a collection of $2^{5-1} = 16$ subsets $S_1, \ldots, S_{16}$ of the set $\{1, \ldots, 5\}$ such that $S_i \cap S_j \neq \emptyset$ and the intersection of all of them (i.e. $\cap_{i=1}^{16} S_i$) is empty. Can you generalize your construction to other values of n? (N.B. It is insufficient to give an arrangement but not give an argument as to why the arrangement works.)
   - We skip Chapter 4 of the text.
   - Read Chapter 5
   - Pages 153 –160 numbers 16, 17, 25

     Also do Pages 153–160 numbers 13, 15, 22, 28, 35, 40, 47. Note that in this section there are numerous typos, including in problems 26, 44 and 45. I haven't assigned these, but you can find the typos in the errata sheet posted on the course webpage or you can try finding them yourself!

4. Due Friday, October 8
   Turn in: Pages 153–160 numbers 30, 36, 37 and 46

5. Due Friday, October 22: Pages 200-205 - number 7

   Also do - 2, 3, 9, 10, 11, 12, 15, 16, 17, 21, 22, 23 (for last identity, can you give a combinatorial proof as well?), 24(a), 26, 29, 31

6. Due Friday, October 29: Pages 200-205 - number 3, 9, 16, 24(a)

7. Due Friday, November 5: Pages 257–264 1(d), 8, 14 (b), 48 (c)

   Also do 1(a), 2, 11, 34, 43, 45, 9, 14(a), 14(c), 16, 27, 38(c), 48(b)

8. Due Friday, November 19: Prove that Fisher's Inequality holds in the case that $\lambda = 1$ without using linear algebra. (Hint: Consider a point x and a block B not containing x and recall the established arithmetic conditions that we have.). Pages 388 – 394 numbers 20, 28, 33,

   Also do: Pages 388 – 394 numbers 20, 21, 28, 31, 32, 33, 35

9. Due Friday, December 3: Pages 388 – 394 numbers 42, 48, 55, 60.
   Read Martin Gardner's *Euler's Spoilers*

   Also do: Pages 388 – 394 numbers 44, 52, 57, 58, 59, 61, 62

10. Due Friday, December 10: Polyá Counting! – Pages 576–581 numbers 10 and 22

    Also do Pages 576–581 numbers 8, 11, 15, 18, 23(a)

11. Final Exam: Tuesday, December 14th, 9am-12pm